華中科技大學数学中心
Center for Mathematical Sciences

中国 武汉
Wuhan China

Newsletter, Summer 2023

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華中科技大學 数学中心
Center for Mathematical Sciences

华中科技大学数学中心简介

在建设世界一流大学的征程中，数学学科的作用异常重要。华中科技大学高瞻远瞩，于2013年成立数学中心。华中科技大学数学中心一方面倡导数学不同分支之间的相互交叉，激发新的合作研究，催生新的研究领域和研究群体。另一方面引领数学与工科、理科，医科及其它学科之间的合作研究，实现交叉创新、 合作共赢。

作为我校国际交流与合作的平台，数学中心大力推动与发展“跨学科应用数学”合作研究。我们的跨学科合作研究领域包括数学与地球科学（物理海洋学和气候动力学）的交叉研究，以及数学与生命科学（计算和定量生物学）的交叉研究。

华中科技大学数学中心积极开展前瞻性研究，立足华中、辐射全国、影响海外。数学中心将国际先进的人才培养模式和研究机构运行机制有机融入到我国建设一流大学与一流学科的伟大事业之中，努力成为培养和聚集一流人才的平台， 国际交流与合作的平台，科教运行机制以及人事体制改革试点的平台。

数学中心成员包括院士，国家特聘专家，外专千人计划专家，长江学者，青年学术英才，楚天学者，洪堡学者和华中学者。还有一批海内外知名访问学者，博士后，博士生，以及来自多个国家的留学生。数学中心设有李国平讲座教授，东湖讲座教授，东湖数学论坛，和郭友中数理科学讲座。

希望重要的数学发现萌芽于此，
希望新的研究领域和研究群体产生于此，
希望著名数学家和科学家在此留下足迹，
希望科技界更深刻地感受到数学的作用：
数学强，则科技强；科技强，则国家强！

数学中心官网

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学术活动

报告题目：Products of random matrices and applications

报告人：Quansheng Liu (Université de Bretagne Sud, France)

时间：2023.04.11，周二，下午 4:00（北京时间）

腾讯会议：324989904

报告摘要：

Some recent progress on limit theorems for products of independent and identically distributed random matrices will be presented. We focus on precise large deviations and convergence rates in the Gaussian approximation. Applications to multi-type branching processes in random environments and branching random walks will also be presented. (Mainly based on a joint work with Ion Grama and Hui Xiao: J. Eur. Math. Soc. 2022)

报告人简介：

- 刘全升，法国南布列塔尼大学特级教授。享受法国优秀科研津贴（PES/PEDR）。1984 和 1987 分别获得武汉大学数学学士和硕士文凭；1993 年 2 月获得巴黎第六大学概率论专业博士文凭（导师是 Y. Guivarc'h 教授）。1993 年至 2000 年任法国雷恩大学讲师、副教授。2000 年 9 月起任法国南布列塔尼大学教授。多次任法国国家基金委评审专家，和波兰国家研究中心基金委评审专家。

  主要研究方向为：概率统计和数字图像处理。近年主要研究随机环境中的概率统计问题，尤其是关于大偏差理论、随机矩阵乘积，几类重要的随机环境的数学物理和应用概率模型，例如分枝过程、分枝随机游动和图像去噪。在 *J. Eur. Math. Soc.*、*Probab. Th. Rel. Fields*、*Annals of Probability*、*Annals of Applied Probability*、*Stochastic Processes and Applications*、*Bernoulli*、*Annals of Inst. Henri Poincaré*、*IEEE Trans. Image Processing*、*SIAM J. Imaging Sciences*、*Inverse Problems and Imaging*、*J. Scientific Computing* 等期刊上发表 100 余篇论文。

报告题目：方阵相似的不变量理论

报告人：许金兴（中国科学技术大学）
时间：2023.04.14，周五，下午 3:00-4:30（北京时间）
线下：松山湖国际创新创业社区 A5 1806 会议室
线上腾讯会议：196452768

报告摘要：

方阵的迹与行列式均为方阵空间上的相似不变函数。一般地，方阵空间上的相似不变函数由特征根的初等对称多项式生成。本报告将从这个初等定理出发介绍代数不变量理论的一些基本概念和基本结果，并介绍上述相似不变函数问题在多个交换方阵时的推广，以及其与 Higgs 丛的 Hitchin 态射的联系。

报告人简介：

- 许金兴于2011年在北京大学获得博士学位，现为中国科学技术大学数学学院教授。研究方向为代数几何，在 Calabi-Yau 簇的模空间、p 进制代数簇的上同调群、代数不变量等问题上得到了一批原创性成果；在 Adv. Math.，Int. Math. Res. Not.等学术期刊上发表论文多篇。

报告题目：Hopf algebra actions on Artin-Schelter regular algebras

报告人：Dingguo Wang 王顶国（曲阜师范大学）
时间：2023.04.21，周五，下午 3:00-4:00（北京时间）
腾讯会议：228849566

报告摘要：

In this talk, we review some recent results of Hopf actions on algebras and related algebras. Then we report the classify all inner-faithful of a non-semisimple Hopf algebra actions on noetherian Koszul Artin-Schelter regular algebras of global dimension up to three. This is based on a joint work with Hui-Xiang Chen and James J. Zhang.

报告人简介：

- 王顶国，曲阜师范大学数学科学学院教授、博士生导师。先后访问过比利时布鲁塞尔自由大学、美国华盛顿大学，从事 Hopf 代数和非交换代数等领域的研究，曾

获第六届山东省青年科技奖和山东省科技进步奖等，主持和参与完成国家自然科学基金多项，在《Trans. Amer. Math. Soc.》、《J. Algebra》、《J. Pure Appl. Algebra》、《Israel J. Math.》等国际著名期刊发表论文 80 余篇。

报告题目：On a General Construction of Optimal Exact Confidence Intervals

报告人：Prof. Weizhen Wang (Wright State University)

时间：2023.06.02，周五，上午 10:00-12:00（北京时间）

腾讯会议：124-666-065

报告摘要：

A general method, named the h-function method, is introduced to unify the constructions of the level-α exact test and 1-α exact confidence interval. Using this method, any given confidence interval can be improved as follows: (i) an approximate interval, including a point estimator, is modified to an exact interval; (ii) an exact interval is refined to be an interval that is a subset of the previous one. Some real datasets, including Johnson & Johnson's Janssen vaccine (2021), are used to illustrate the method. Senior students in Statistics have the background to access the topics.

数学中心短课

华中科技大学数学中心于 4-6 月成功举办多项短课活动。

1. SDE + Lévy Processes

短课演讲人：**袁胜兰（Institut für Mathematik, Universität Augsburg）**

时间：上午 9:00-11:00

地点：华中科技大学恩明楼 813

	具体日期	详细内容
1	2023.04.22	Lévy Processes
2	2023.04.23	Lévy-Itô Decomposition
3	2023.04.29	Stochastic Integrals
4	2023.04.30	Itô's Formula
5	2023.05.06	SDE with Semimartingale
6	2023.05.07	Continuity and Differentiability
7	2023.05.13	Homeomorphic Property
8	2023.05.14	Stochastic Flow of Diffeomorphisms

参考文献：

Hiroshi Kunita, Stochastic Differential Equations Based on Lévy Processes and Stochastic Flows of Diffeomorphisms

报告人简介：

- 袁胜兰：助理研究员。2017 年 9 月至 2018 年 8 月前往德累斯顿工业大学 CSC 联合培养博士。2019 年 6 月获华中科技大学概率论与数理统计专业博士学位。随后加入华中科技大学人工智能与自动化学院从事博士后研究。而后任德国奥格斯堡大学助理研究员职位。研究方向为 Lévy 过程驱动的随机动力系统、量子力学、统计物理和随机分析。近五年在 SIAM Journal on Applied Dynamical Systems、Journal of Statistical Mechanics、Analysis and Applications 等国际重要期刊上发表 14 篇学术论文。

2. Exact Statistical Inference and Applications in Biostatistics

2023 年 5 月美国莱特州立大学王维真教授讲授“Exact Statistical Inference and Applications in Biostatistics”短课。

Outline：

Introduction

Approximate and exact inferences

Exact tests and intervals under the normal model

Exact tests for intervals based on pivotal quantities

Approximate tests and intervals

Are approximate tests and intervals reliable?

The h-function method

Inferences for a proportion p in a binomial distribution;

Inferences for M or N in a hypergeometric distribution

Inferences based on non-minimum sufficient statistic

Inferences for comparison of two proportions with two independent binomials

Inferences for comparison of two proportions in a matched pairs experiment

Inferences in comparison of two proportions using combined data

Inferences in a 2×2 table with zero structural.

Inferences for comparison of two proportions in two matched pairs experiments

Comparison of two raters

Biography:

Weizhen Wang received his B.S. and M.S. at Peking University in 1987 and 1990, respectively, and completed his Ph.D. in Statistics at Cornell University in 1995. After the one-year visit at Purdue University, he joined Wright State University and has been a Professor of Statistics since 2007. His research includes bioequivalence, exact parametric and nonparametric inferences, saturated and adaptive designs, categorical data analysis, the foundation of statistics, statistical computation, and clinical trials. His current primary interest is exact statistical inference and the implementation in R.

3. Introduction to Number Theory

Speaker: Prof. Daqing Wan (University of California, Irvine)

Tencent ID: 631-8867-6913 Password: 036586

Abstract:

This will be a self contained introduction to basic number theory. Some emphasis will be given to those topics which are useful in mathematical cryptography, including finite fields, number fields, p-adic fields, and polynomial arithmetic on those fields.

Schedule:

	Date	China Time
1	2023.06.03	7:30-9:30 (P.M.)
2	2023.06.04	7:30-9:30 (P.M.)
3	2023.06.05	7:30-9:30 (P.M.)
4	2023.06.06	7:30-9:30 (P.M.)
5	2023.06.07	7:30-9:30 (P.M.)
6	2023.06.08	7:30-9:30 (P.M.)

Biography:

Daqing Wan is a professor of mathematics at the University of California at Irvine. His research interests include number theory, arithmetic geometry, algorithms and complexity. He has published over 100 research papers in mathematics and computer science.

4. Computational Number Theory

Speaker: Prof. Qi Cheng (University of Oklahoma)

Tencent ID: 587-7079-8858 Password: 75530

Topics:

- Basics of computational complexity
- Basics of computational number theory, including Extended Euclidean algorithm, Fermat Little Theorem, repeated squaring algorithm, Chinese Remainder Theorem and finite fields.
- Primality test, integer factorization and discrete logarithm over finite fields.
- Computational lattice theory, number fields and ideal lattices (if time permits).

Schedule:

	Date	China Time
1	2023.06.09	3:00-5:00 (P.M.)
2	2023.06.09	7:30-9:30 (P.M.)
3	2023.06.10	9:00-11:00 (A.M.)
4	2023.06.10	3:00-5:00 (P.M.)
5	2023.07.02	9:00-11:00 (A.M.)
6	2023.07.04	9:00-11:00 (A.M.)
7	2023.07.06	9:00-11:00 (A.M.)
8	2023.07.10	9:00-11:00 (A.M.)
9	2023.08.08	9:00-11:00 (A.M.)
10	2023.08.10	9:00-11:00 (A.M.)

Biography:

Qi Cheng is now a professor in the School of Computer Science at the University of Oklahoma. He received his PhD in Computer Science from University of Southern California in 2001. His research interests are in the areas of cryptography, computational number theory, coding theory, computational complexity, algorithmic self-assembly and DNA computing. He has published over 30 research articles in journals and conference proceedings.

更多课程回放内容，关注哔哩哔哩官方账号：华中大数学中心

国家天元数学中部中心随机动力系统会议

报到时间：2023 年 6 月 14 日　14:00-18:00
报到地点：武汉君宜王朝大饭店
会议时间：　2023 年 6 月 15-18 日
会议地点：武汉君宜王朝大饭店 6 楼绿岛厅　　　　腾讯会议号：922-6078-0232

学术委员会（按姓氏排序）：

段金桥	华中科技大学
高洪俊	东南大学
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吕克宁	Brigham Young University
王伟	南京大学
吴奖伦	北师大浸会联合国际学院
吴付科	华中科技大学
赵怀忠	Durham University
周盛凡	浙江师范大学

会议组委会：

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刘显明	华中科技大学
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张奥	中南大学
陈小丽	新加坡国立大学
陈涌	浙江理工大学
乔会杰	东南大学

该会议旨在通过促进随机和计算新兴领域共同感兴趣的主题，在随机和计算社区之间建立桥梁，并教育博士后和研究生在这些新兴领域进行研究。

会议目标：

本次会议将深入讨论随机动力系统、随机分析与计算及在数据科学热点问题中的新研究。将聚焦随机偏微分方程、随机动力系统的理论方法及其在生物物理中的应用，随机动力系统的有效动力学与数据科学交叉研究的前沿问题，融合机理的数据驱动建模方法，深度学习的可解释性分析，因果推断，复杂网络，隐私计算等。邀请相关专家介绍最新研究成果，探讨与之适应的新的建模技术与分析方法，为解决诸多复杂应用问题提供新的研究思路与方向。

主要内容：

为了更好地理解复杂系统，考虑非线性和随机性是必不可少的。人们越来越认识到将随机效应纳入科学和工程中复杂、多尺度现象的建模中的作用。考虑随机效应对于解决当今数据科学中的许多问题至关重要。

1. 随机偏微分方程

随机偏微分方程作为受到随机扰动的复杂系统的数学模型，广泛存在于气候系统、生物物理学、凝聚态物理学、材料科学、信息系统、机械和电气工程以及金融中的复杂现象的数学建模中。我们研究随机偏微分方程的渐近动力学行为，特别是不变流形、不变测度及其数值实现，以及通过平均化和均匀化的有效约化。不变测度是承载非线性动力系统基本动力信息的分析对象。例如，使用不变的度量，我们可以计算系统状态的整体平均值（熵、平均状态、矩、可预测性等），我们还可以通过平均来减少动态。

2. 随机动力系统的理论方法及其在生物物理中的应用

随机微分方程作为非线性系统在随机因素影响下的合适的数学模型，已得到广泛接受。随机微分方程及其所生成的随机动力系统的复杂动力学的研究目前已经是国内外微分方程及动力系统领域的研究热点和前沿课题。随机动力系统的理论方法在近些年得到了迅速发展，几何方法，量化指标等等。将这些方法应用到生物物理系统中，借助具有可预测功能的动力系统模型做出定量预测，进而通过已有实验数据分析随机系统的信息，为生物物理学的随机动力学研究提供新方法和新工具。

3. 融合机理的数据驱动建模方法与可解释性的深度学习

随着数据采集技术的提升，越来越多的复杂时空数据对建模和应用的分析方法提出挑战。考虑到时空数据的强噪声、复杂突变、难以预测、观测不完全等特性，需要融合机理建模，从而更好地解释数据动力学的内在本质。这里涉及到的数学工具涉及面广：泛函分析、逼近论、随机分析、概率论、数值分析、最优化、控制论、动力系统、几何拓扑、统计推断等；还有这其中不同领域方法的交叉，于是许多新的技术方法不断产生，并激发我们探索更多创新性理论方法。深度学习的可解释性有助于揭示数据科学中的重大瓶颈问题，对于许多新领域如因果推理、复杂网络、隐私计算等有着重大帮助。

数学中心优秀毕业生

Larissa Serdukova（杨甜），于 2013 年 9 月-2017 年 7 月在华中科技大学数学中心学习，2017 年获华中科技大学概率论与数理统计专业博士学位，师从段金桥教授。2017 年 8 月-2020 年 9 月在 Georgia Institute of Technology（美国佐治亚理工大学）从事博士后研究，2020 年 10 月-2022 年 8 月在 University of Reading（英国雷丁大学）做科研助理。现在为英国 University of Leicester（莱斯特大学）助理教授。研究方向包括随机微分方程，随机动力系统，非光滑动力系统，亚稳现象，非高斯 Lévy 运动驱动的动力系统，振动能量收集系统及气候动力学。近年在 Chaos、Nonlinear Dynamics、Nonlinear Processes in Geophysics、International Journal of Mechanical Sciences、Scientific reports 等国际重要期刊上发表 13 篇学术论文。

Entrance Exams 入学考试

微分几何

华中科技大学数学中心

Summer 2023

Total: 100 points (20 points for each problem).

1. 设 v_1,v_2,v_3,v_4 是 $\mathcal{R}^3$ 的四个向量，证明：
   （1）$v_1\wedge(v_2\wedge v_3)=<v_1,v_3>v_2-<v_1,v_2>v_3$；
   （2）Lagrange 恒等式
   $<v_1\wedge v_2,v_3\wedge v_4>=<v_1,v_3><v_2,v_4>-<v_1,v_4><v_2,v_3>$；
   （3）$(v_1,v_2,v_3)=(v_2,v_3,v_1)=(v_3,v_1,v_2)$.

2. （1）求曲线 $y=ax^2$ 的弧长与曲率；
   （2）求曲线 $\mathbf{r}(t)=(3t-t^2,3t^2,3t+t^2)$ 的曲率与挠率。

3. （1）证明：曲面 $F(\frac{y}{x},\frac{z}{x})=0$ 的任意切平面过原点；
   （2）求曲面 $\mathbf{r}(u,\upsilon)=(a(u+\upsilon),b(u-\upsilon),4u\upsilon)$ 的 Gauss 曲率、平均曲率、主曲率及对应的主方向。

4. 球面 $\mathbf{r}=(a\cos u\cos\upsilon,a\cos u\sin\upsilon,a\sin u)$，
   （1）求球面的一组正交活动标架；
   （2）求相应的诸微分形式 $\{\omega_1,\omega_2,\omega_{12},\omega_{13},\omega_{23}\}$；
   （3）求球面的第二基本形式 II.

5. 在球面 $\mathbf{r}=(a\cos u\cos\upsilon,a\cos u\sin\upsilon,a\sin u)$ 上，
   （1）证明：曲线的测地曲率可以表示为
   $$k_g=\frac{d\theta}{ds}-\sin u\frac{d\upsilon}{ds},$$
   其中 s 是曲线 $(u(s),\upsilon(s))$ 的弧长参数，θ 是曲线与经线（u 线）的夹角；
   （2）求球面纬圆的测地曲率。

概率论

华中科技大学数学中心

Total: 100 points. (Each problem: 20 points)

以下问题中的概率空间里的 sigma-域为 F，概率测度为 P

1. 假设 X: Ω→R 是一个取实值的随机变量，函数 f: R→R 是一个 Borel 可测函数。证明: f(X)也是一个取实值的随机变量。

2. 假定 X, Y 是两个独立的正态随机变量，且 X~N(0,1), Y~N(0,1)。分别计算（X,Y）和－X 的概率密度函数。

3. 设随机变量序列 X_n (n=1,2,⋯)相互独立且服从同一标准均匀分布 U（0,1）。求 X_n 的特征函数。考虑 $\eta_n = \sum_{k=1}^{n} X_k$，并求 η_n 的特征函数。

4. 设随机变量序列{X_n}均方收敛于 X。试问：X_n 是依概率收敛于 X 吗？若是，请证明之；若不是，请举出反例。

5. 随机变量序列 X_n, Y_n 在 L^2 意义下分别收敛于 X, Y。试问：在 L^1 意义下，X_nY_n 是否收敛到 XY？若是，请证明；若不是，请举出反例。
   (提示：用 Cauchy-Schwarz 不等式)

常微分方程与动力系统

华中科技大学数学中心

2023 年 6 月

1. 请作出系统

$$\frac{dx}{dt}=-x-y,\ \frac{dy}{dt}=x-3y$$

的相图。

2. 求微分方程组

$$\frac{dy}{dx}=\mathrm{A}\mathrm{y}+\mathrm{f}(x)$$

满足初值条件 $\mathrm{y}(0)=\eta$ 的解，其中

$$\mathrm{A}=\begin{pmatrix}4 & -3\\ 2 & -1\end{pmatrix},\ \mathrm{f}(x)=\begin{pmatrix}\sin x\\ -2\cos x\end{pmatrix},\ \eta=\begin{pmatrix}0\\ 0\end{pmatrix}.$$

3. 设函数 $P(x,y)$ 和 $Q(x,y)$ 在单连通区域 D 内连续可微，且

$$\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}\neq 0,\ (x,y)\in D.$$

试证系统

$$\dot{x}=P(x,y),\ \dot{y}=Q(x,y)$$

在 D 内不存在闭轨线。

4. 设质点 M 在平面内运动，其运动方程在极坐标系下表达为：

$$\begin{cases}\dfrac{dr}{dt}=r(r^2-1)\\ \dfrac{d\theta}{dt}=1.00000001-\cos^2(\theta).\end{cases}$$

(1) 请画出该系统的相图；(2) 解释该质点的运动规律。

5. 求下列方程的通解：

(1) $\ddot{x}+\dot{x}-2x=8\sin 2t$；

(2) $t^2\ddot{x}-4t\dot{x}+6x=0$.

6. 设 $x\in\mathbb{R}^1$，函数 $g(x)$ 连续，且当 $x\neq 0$ 时有 $xg(x)>0$。试证明方程

$$\ddot{x}+g(x)=0$$

的零解是稳定的，但不是渐进稳定的。

7. 讨论二维微分方程

$$\dot{x}=y \qquad \dot{y}=-1+x^2$$

的两个平衡点的稳定性。

8. 判断下列方程的奇点(0, 0) 的类型，并作出该奇点附近的相图：

(1) $\dot{x}=y-2x+4y+\sin y,\ \dot{y}=x+y+e^y-1$

(2) $\dot{x}=x(1-y),\ \dot{y}=y(1-x)$.

9. 构造 Lyapunov 函数来判断如下系统零解的稳定性：

$$\begin{cases}\dot{x}=y-x^2+3y^2-2xy,\\ \dot{y}=x-y+xy.\end{cases}$$

10. 考虑从 $\mathbb{R}^2$ 到 $\mathbb{R}^2$ 的一个函数 φ_t：

$$\varphi_t(x)=\begin{pmatrix}\varphi_{1t}(x)\\ \varphi_{2t}(x)\end{pmatrix}=\begin{pmatrix}x_1e^{-t}\\ x_2e^{x_1(e^{-t}-1)}\end{pmatrix}$$

证明 φ_t 是一个可微动力系统。

Celebrity Stories 名人故事

Ngô Bảo Châu（吴宝珠）—— The 2010 Fields Medal Winner

Ngô Bảo Châu（吴宝珠），越南-法国数学家，因证明朗兰兹纲领的基本引理（Fundamental Lemma）而知名，于 2010 年被授予菲尔兹奖，也被美国《时代》周刊列为 2009 年十大科学发现之一。目前是美国芝加哥大学数学教授、越南高等数学研究所主任。

1972 年 6 月，吴宝珠出生在越南北部河内的一个学者家庭。他的父亲 Ngô Huy Cân 教授是越南国家力学研究所的物理学教授。他的母亲 Trân Luu Vân Hiên 是河内国立传统医院的副教授。吴宝珠是家里唯一的孩子。由于父亲在苏联获得应用数学博士学位并长期在苏联工作，吴宝珠的童年是在母亲家里度过的。父亲回到越南时，吴宝珠已经开始上小学了。吴宝珠的父亲对他影响很大。吴宝珠曾在讲武实验小学学习，这所小学采用了鼓励自主阅读和言论自由等特殊的教学方法。然而，他的父亲不喜欢这所学校，并把他送到一所针对有数学天赋的学生的天才学校。从那时起，因为他父母的原因，他做了很多数学练习，开始喜欢上数学。

吴宝珠就读于征王初中的特殊班，那里的学生是通过入学考试特别挑选的。1987 年初中毕业后，吴宝珠被河内国立自然科学大学天才高中的一个班录取，该班面向天才学生专门研究数学。吴宝珠在高中的两年时间里参加了第 29 届和第 30 届国际数学奥林匹克竞赛（IMO），并连续获得两枚金牌。第一次更是获得了满分 42 分。

在越南，获得国际数学奥林匹克竞赛的金牌是一种特殊的荣誉。此外，国际数学奥林匹克竞赛的奖牌获得者还可以获得奖学金，到苏联或东欧国家的大学学习。吴宝珠获得了匈牙利政府提供的奖学金。1989 年高中毕业后，由于对组合数学的热爱，他准备去匈牙利留学。然而，在东欧共产主义垮台后，匈牙利新政府停止向来自越南的学生提供奖学金。因为这一意外事件，吴宝珠失去了这个机会。

“我决定研究朗兰兹纲领纯属巧合。我想做点什么，在那个伟大的时代，这是一个很好的决定。”

就在那个时候，一位来自法国的教授访问了吴宝珠父亲工作的力学研究所。在得知吴宝珠获得国际数学奥林匹克竞赛金牌后，教授试图帮助他获得法国政府的奖学金。

吴宝珠的祖父曾在法国留学，开始教他法语。法国的教育体系与其他国家不同。

高中两年后，他去巴黎高等师范学校读本科。他的导师 Michel Broue（迈克•布鲁意）建议他跟随巴黎第十一大学的 Gérard Laumon（热拉尔•洛蒙）教授。所以他在大学里就开始了博士研究。

当时朗兰兹纲领在法国数学届中是一个著名的项目。被称为法国自守形式之父的数学家 Roger Godement（罗杰•戈德门特），将朗兰兹纲领和自守形式引入法国。这对包括洛蒙教授在内的法国数学家产生了很大的影响。在洛蒙教授的建议下，吴宝珠于 1993 年开始研究朗兰兹纲领。

1997 年，吴宝珠在 25 岁的时候获得了巴黎第十一大学的博士学位。吴宝珠于 1998 年成为巴黎北部大学国家科学研究中心（CNRS）的成员。这是他的第一份工作。当时，他的目标是证明朗兰兹纲领的基本引理。法国的培训体系与美国大不相同。在美国，必须在获得博士学位后做两到三年的博士后。在申请工作之前，发表论文的压力很大，即使在找到工作之后，压力仍然存在。然而在法国，吴宝珠没有这种压力，不需要发表论文，他所需要做的就是研究数学。在获得博士学位后的头七年里，吴宝珠是一名研究员，而不是教授。一开始吴宝珠和洛蒙教授一起工作。当他回到基本引理的问题上时，他尝试了不同的方法，得到了新的想法。

吴宝珠在法国很开心，因为他可以专注于数学。“CNRS 类似于中国科学院。研究人员隶属于 CNRS。也就是说，CNRS 给他们发工资；虽然研究人员与相关大学的教授一起工作，但他们不一定要教书。我不知道这样的安排是好是坏，但我获得博士学位后的那段时间对我来说真的是一个黄金时代。当我成为 CNRS 的研究员，这是一个终身的职位。我没有申请基金、发表论文、担心终身职位或教书的压力。我所需要做的就是留在那里，花更多的时间在数学研究上，而不是做其他事情。”

根据美国数学会 MathSciNet 的统计，吴宝珠到目前为止已经发表了 15 篇论文。他说:“我对发表低质量的论文不感兴趣。我写的论文很少，但都很好。我的同事告诉我，不要浪费时间写糟糕的论文。一篇好论文胜过一百篇质量差的论文。这不是我的方式，而是法国的标准。”如果他没有发表论文，同事们如何评价他的工作?“我每年都接受评估。我只需要报告我在这一年中所做的事情。法国国家研究委员会每五年对我进行一次评估。我报告我已经取得的成就和我打算做什么。如果他们觉得还不错，并给我一个好的评价，CNRS 会继续支持我。”

2003 年是一个转折点。吴宝珠说:“那时，我对每一个与几何有关的问题都非常清楚。事情变得简单明了。我相信我有了一个新想法，但这仅仅是个开始。”那年夏天，洛蒙教授应吴宝珠的邀请去越南旅行。洛蒙对吴宝珠的想法产生了兴趣，他们一起成功地证明了酉群的基本引理。2004 年两人因此获得了克雷研究奖。

2005 年，33 岁的他获得了巴黎第十一大学的教授头衔，成为越南有史以来最年轻的正教授。

2006 年，吴宝珠应邀访问普林斯顿大学高等研究院（IAS）。大约在 2006 年 12 月，在与 IAS 的马克•戈瑞斯基（Mark Goresky）的一次谈话中，他受到了启发，吴宝珠相信可以证明一般情形下朗兰兹纲领的基本引理。朗兰兹纲领吸引了吴宝珠。他花了将近 17 年的时间来研究它。

吴宝珠指出:“每个数学家都知道朗兰兹纲领的重要性。如果你知道朗兰兹纲领，你将以一种新的方式理解数学和几何。安德鲁 • 怀尔斯用朗兰兹纲领的思想证明了费马大定理。你可以看到这个纲领是多么的美丽和强大。这真的很令人兴奋。”

2007 年 6 月，吴宝珠完成了论文的初稿，共 200 页。然后，他在法国举行的一个研讨会上就他的证明发表了演讲。“有些人怀疑它的有效性，但大多数人都被我的证据说服了。”回到普林斯顿后，吴宝珠继续在其它会议上发表报告。

到 2009 年底，该领域几乎所有人都认为吴宝珠已经证明了这个猜想。“基本引理”被《时代》杂志列为 2009 年十大科学发现之一。过去的几年里，在巴黎第十一大学和普林斯顿高等研究院工作的越南数学家吴宝珠对这个基本引理进行了巧妙的证明。当它在今年被检验并被证实是正确的时候，全世界的数学家都松了一口气。过去三十年来，数学工作者在这一领域的工作是基于这样一个原则：基本引理确实是准确的，总有一天会被证明。“这就好像人们在河对岸工作，等着有人把这座桥架好，”IAS 的数论学家 Peter Sarnak 说。“现在，突然之间，河对岸每个人的工作都得到了证明。”

2010 年 1 月，吴宝珠的论文“The Fundamental Lemma for Lie Algebras”被法国《高等科学研究所数学出版物》接受并发表。

“吴宝珠是这个时代最伟大的数学家之一，他很聪明。我真的希望这个年轻人能做更多伟大的事情。”——Robert Fefferman，芝加哥大学物理科学系主任，数学系教授。

当然，还有一个人对基本引理的证明非常兴奋，他就是罗伯特 • 朗兰兹，他曾经离开过这个领域，但现在又回来了。2010 年吴宝珠与朗兰兹合作发表了一篇论文。2010 年 1 月，吴宝珠加入芝加哥大学数学系任正教授。

芝加哥大学数学系主任康斯坦丁对吴宝珠有这样的评价。“他证明了一个基本理论，一个被称为基本引理的猜想。它之所以这样命名，是因为它是开启朗兰兹纲领进程的钥匙……吴宝珠的证明戏剧性地打开了这扇门。”

2010 年在印度海得拉巴举行的第 26 届国际数学家大会上，吴宝珠因其在 2008 年证明了朗兰兹纲领的基本引理而被授予菲尔兹奖。朗兰兹纲领是加拿大裔美国数学家罗伯特 • 朗兰兹提出的。1979 年，他提出了一个雄心勃勃的革命性理论，将数论和群论这两个数学分支联系起来。在一系列令人眼花缭乱的猜想和见解中，这个理论抓住了与涉及整数的方程相关的深层对称性，奠定了现在被称为朗兰兹纲领的基础。朗兰兹知道，要证明其理论基础的假设，需要几代人的努力。然而，他确信证明所有这些

首先需要一个垫脚石——基本引理。他，他的合作者和他的学生能够证明基本引理的特殊情况。然而，证明一般情况比朗兰兹预期的要困难得多——以至于花了 30 年的时间才最终实现。吴宝珠最终在 2008 年通过他的新方法证明了引理。

“参加好的研讨会是非常重要的，与他人保持交流是必要的。当我第一年参加研讨会时，我一个字也听不懂，但我坚持听。”

对于吴宝珠来说，从数学奥林匹克奖牌获得者成为数学家并不容易。并不是所有的奥数获奖者长大后都是数学家，但在越南，几乎所有的数学家都是奥数奖牌获得者。

回顾他的数学之旅，吴宝珠说:“参加奥数和研究数学是不同的。参加奥数需要掌握各种技能，这些技能将有助于在有限的时间内解决复杂而高水平的问题。这样做的危险在于，人们可能不尊重数学自然的简洁性和美。一个人能否成为数学家，最终取决于他本人和他欣赏数学的能力。这种转变既不直接也不明显。在我看来，要想成为一名优秀的数学家，必须成为一名 ‘数学鉴赏家’。”一个人怎样才能培养自己对数学的兴趣呢?“要做到这一点，需要花费大量时间在数学上，学习和了解更多的数学知识。

吴宝珠向刚开始学习数学的学生建议说:“在法国，学生必须参加许多基础课程和富有成效的讨论。在大学期间，你可以通过参与讨论来培养良好的品位。你可以从优秀数学家的演讲中了解数学家是如何提出问题的，他们为什么对这些问题感兴趣，他们是如何讨论这些问题的，以及如何证明这些问题的。我有幸参与了许多讨论和项目，并从中学到了很多东西。当我还是研究生的时候，我就提出了这个问题的证明。如果我没有参加讨论，我就无法自己解决问题，因此也就无法完成这个项目。”在谈到数学时，他说:“当你想研究数学时，从事数学工作是令人愉快的。你会以最自然的形式感受到它；数学是描述世界最美丽的语言。它非常简单，因此也是最实用的语言。它准确而简洁。”

在谈到未来时，吴宝珠说:“我只证明了基本引理，而不是纲领中的所有内容。我们的下一个目标是朗兰兹纲领，基本引理只是它的基础，就像一座小山。登上这座山，我们就能看到朗兰兹纲领的全貌。我们面前有一座大山，但现在的问题是如何攀登它。朗兰兹回来了，这是一件好事，他将为我们指明解决问题的新方向。我想我可能要用一生的时间来证明这个纲领中的一切。”

Popular Mathematics 数学热门话题

Physicists Create Elusive Particles That Remember Their Pasts

物理学家创造了难以捉摸的粒子，它们能记住自己的过去

在两个具有里程碑意义的实验中，研究人员使用量子处理器来设计几十年来一直吸引着物理学家的奇异粒子。这项工作是向防碰撞量子计算机迈出的一步。通过将粒子彼此“编织”在一起，量子计算机可以以一种防止错误的方式存储和操纵信息。

Forty years ago, Frank Wilczek was mulling over a bizarre type of particle that could live only in a flat universe. Had he put pen to paper and done the calculations, Wilczek would have found that these then-theoretical particles held an otherworldly memory of their past, one woven too thoroughly into the fabric of reality for any one disturbance to erase it.

However, seeing no reason that nature should allow such strange beasts to exist, the future Nobel prize-winning physicist chose not to follow his thought experiments to their most outlandish conclusions — despite the objections of his collaborator Anthony Zee, a renowned theoretical physicist at the University of California, Santa Barbara.

“I said, ‘Come on, Tony, people are going to make fun of us,’” said Wilczek, now a professor at the Massachusetts Institute of Technology.

Others weren’t so reluctant. Researchers have spent millions of dollars over the past three decades or so trying to capture and tame the particlelike objects, which go by the cryptic moniker of non-abelian anyons.

Now two landmark experiments have finally succeeded, and no one is laughing. “This has been a target, and now it’s hit,” Wilczek said.

Physicists working with the company Quantinuum announced today that they had used the company’s newly unveiled, next-generation H_2 processor to synthesize and manipulate non-abelian anyons in a novel phase of quantum matter. Their work follows a preprint posted last fall in which researchers with Google celebrated the first clear intertwining of non-abelian objects, a proof of concept that information can be stored and manipulated in their shared memory. Together, the experiments flex the growing muscle of quantum devices while offering a potential glimpse into the future of computing: By maintaining nearly indestructible records of their journeys through space and time, non-abelian anyons could offer the most promising platform for building error-tolerant quantum computers.

“As pure science, it’s just, wow,” said Ady Stern, a condensed matter theorist at the Weizmann Institute of Science in Israel who has spent his career studying the objects. “This brings you closer [to topological quantum computing]. But if there’s one thing the last few decades have shown us, it’s a long and winding road.”

Flatland Computing

In 1982, Wilczek helped open physicists’ minds to the menagerie of particles that could exist in two dimensions. He worked out the consequences of confining quantum laws to a hypothetical, entirely flat universe, and found that it would contain strange particles with fractional spins and charges. Moreover, swapping otherwise indistinguishable particles could change them in ways that were impossible for their three-dimensional counterparts. Wilczek cheekily named these two-dimensional particles anyons, since they seemed to be capable of nearly anything.

Wilczek focused on the simplest “abelian” anyons, particles that, when swapped, change in subtle ways that are not directly detectable.

He stopped short of exploring the wilder option — non-abelian anyons, particles that share a memory. Swapping the positions of two non-abelian anyons produces a directly observable effect. It switches the state of their shared wave function, a quantity that describes a system’s quantum nature. If you stumble upon two identical non-abelian anyons, by measuring which state they are in, you can tell whether they have always been in those positions or whether they’ve crossed paths — a power no other particle can claim.

To Wilczek, that notion seemed too fantastical to develop into a formal theory. “What kinds of states of matter support those?” he recalled thinking.

But in 1991, two physicists identified those states. They predicted that, when subjected to strong enough magnetic fields and cold enough temperatures, electrons stuck to a surface would swirl together in just the right way to form non-abelian anyons. The anyons would not be fundamental particles — our 3D world forbids that — but “quasiparticles.” These are collections of particles, but they are best thought of as individual units. Quasiparticles have precise locations and behaviors, just as collections of water molecules produce waves and whirlpools.

In 1997, Alexei Kitaev, a theorist at the California Institute of Technology, pointed out that such quasiparticles could lay the perfect foundation for quantum computers. Physicists have long salivated at the possibility of harnessing the quantum world to perform calculations beyond the reach of typical computers and their binary bits. But qubits, the atomlike building blocks of quantum computers, are fragile. Their wave functions collapse at the lightest touch, erasing their memories and their ability to perform quantum calculations. This flimsiness has complicated ambitions to control qubits long enough for them to finish lengthy calculations.

Kitaev realized that the shared memory of non-abelian anyons could serve as an ideal qubit. For starters, it was malleable. You could change the state of the qubit — flipping a zero to a one — by exchanging the positions of the anyons in a manner known as “braiding.”

You could also read out the state of the qubit. When the simplest non-abelian anyons are brought together and “fused,” for instance, they will emit another quasiparticle only if they have been braided. This quasiparticle serves as a physical record of their crisscrossed journey through space and time.

And crucially, the memory is also nigh incorruptible. As long as the anyons are kept far apart, poking at any individual particle won’t change the state the pair is in — whether zero or one. In this way, their collective memory is effectively cut off from the cacophony of the universe.

“This would be the perfect place to hide information,” said Maissam Barkeshli, a condensed matter theorist at the University of Maryland.

Unruly Electrons

Kitaev's proposal came to be known as "topological" quantum computing because it relied on the topology of the braids. The term refers to broad features of the braid — for example, the number of turns — that aren't affected by any specific deformation of their path. Most researchers now believe that braids are the future of quantum computing, in one form or another. Microsoft, for instance, has researchers trying to persuade electrons to form non-abelian anyons directly. Already, the company has invested millions of dollars into building tiny wires that — at sufficiently frigid temperatures — should host the simplest species of braidable quasiparticles at their tips. The expectation is that at these low temperatures, electrons will naturally gather to form anyons, which in turn can be braided into reliable qubits.

After a decade of effort, though, those researchers are still struggling to prove that their approach will work. A splashy 2018 claim that they had finally detected the simplest type of non-abelian quasiparticle, known as "Majorana zero modes," was followed by a similarly high-profile retraction in 2021. The company reported new progress in a 2022 preprint, but few independent researchers expect to see successful braiding soon.

Similar efforts to turn electrons into non-abelian anyons have also stalled. Bob Willett of Nokia Bell Labs has probably come the closest in his attempts to corral electrons in gallium arsenide, where promising but subtle signs of braiding exist. The data is messy, however, and the ultracold temperature, ultrapure materials, and ultrastrong magnetic fields make the experiment tough to reproduce.

"There has been a long history of not observing anything," said Eun-Ah Kim of Cornell University.

Wrangling electrons, however, is not the only way to make non-abelian quasiparticles.

"I had given up on all of this," said Kim, who spent years coming up with ways to detect anyons as a graduate student and now collaborates with Google. "Then came the quantum simulators."

Compliant Qubits

Quantum processors are changing the hunt for anyons. Instead of trying to coax hordes of electrons to fall into line, in recent years researchers have begun using the devices to bend individual qubits to their will. Some physicists consider these efforts simulations, because the qubits inside the processor are abstractions of particles (while their physical nature varies from lab to lab, you can visualize them as particles spinning around an axis). But the quantum nature of the qubits is real, so — simulations or not — the processors have become playgrounds for topological experiments.

“It breathes new life” into the field, said Fiona Burnell, a condensed matter theorist at the University of Minnesota, “because it’s been so hard to make solid-state systems.”

Synthesizing anyons on quantum processors is an alternate way to leverage the power of Kitaev’s braids: Accept that your qubits are mediocre, and correct their errors. Today’s shoddy qubits don’t work for very long, so anyons built from them would also have short lifetimes. The dream is to quickly and repeatedly measure groups of qubits and correct errors as they crop up, thereby extending the life span of the anyons. Measurement erases an individual qubit’s quantum information by collapsing its wave function and turning it into a classical bit. That would happen here too, but the important information would remain untouchable — hidden in the collective state of many anyons. In this way, Google and other companies hope to shore up qubits with fast measurements and swift corrections (as opposed to low temperatures).

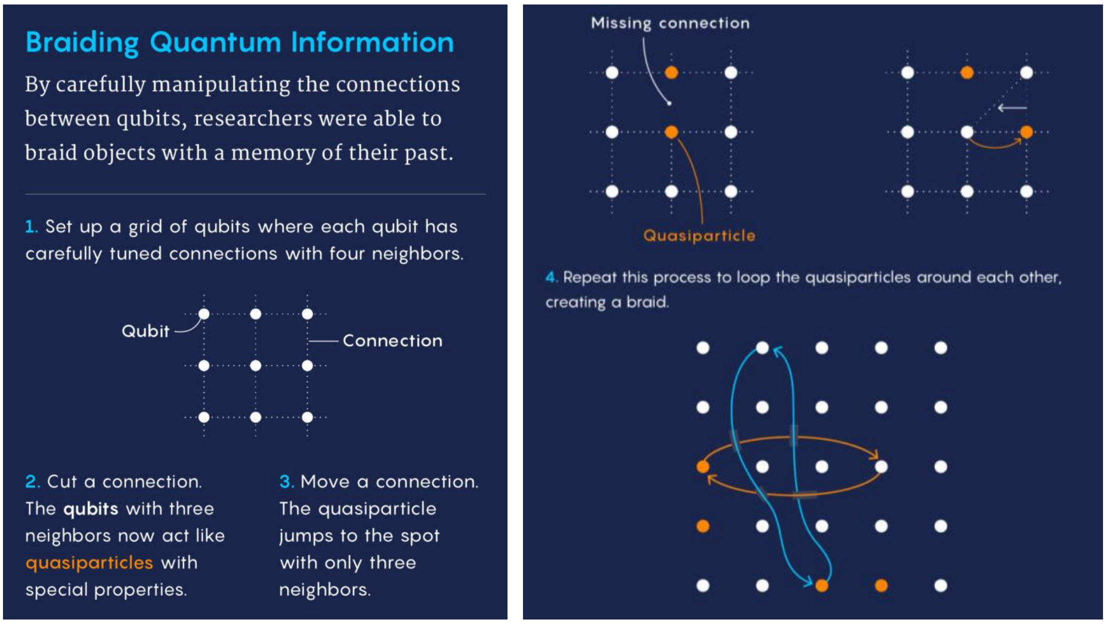
“Ever since Kitaev,” said Mike Zaletel, a condensed matter physicist at the University of California, Berkeley, “this has been the way people think quantum error correction will likely work.”

Google took a major step toward quantum error correction in the spring of 2021, when researchers assembled about two dozen qubits into the simplest grid capable of quantum error correction, a phase of matter known as the toric code.

Creating the toric code on Google’s processor amounts to forcing each qubit to strictly cooperate with its neighbors by gently nudging them with microwave pulses. Left unmeasured, a qubit points in a superposition of many possible directions. Google's processor effectively cut down on those options by making each qubit coordinate its spin axis with its four neighbors in specific ways. While the toric code has topological properties that can be used for quantum error correction, it doesn't natively host non-abelian quasiparticles. For that, Google had to turn to a strange trick long known to theorists: certain imperfections in the grid of qubits, dubbed "twist defects," can acquire non-abelian magic.

Last fall, Kim and Yuri Lensky, a theorist at Cornell, along with Google researchers, posted a recipe for easily making and braiding pairs of defects in the toric code. In a preprint posted shortly after, experimentalists at Google reported implementing that idea, which involved severing connections between neighboring qubits. The resulting flaws in the qubit grid acted just like the simplest species of non-abelian quasiparticle, Microsoft's Majorana zero modes.

"My initial reaction was 'Wow, Google just simulated what Microsoft is trying to build. It was a real flexing moment," said Tyler Ellison, a physicist at Yale University.



Braiding Quantum Information

By carefully manipulating the connections between qubits, researchers were able to braid objects with a memory of their past.

1. Set up a grid of qubits where each qubit has carefully tuned connections with four neighbors.

2. Cut a connection. The **qubits** with three neighbors now act like quasiparticles with special properties.

3. Move a connection. The quasiparticle jumps to the spot with only three neighbors.

4. Repeat this process to loop the quasiparticles around each other, creating a braid.

By tweaking which connections they cut, the researchers could move the deformations. They made two pairs of non-abelian defects, and by sliding them around a five-by-five-qubit chessboard, they just barely eked out a braid. The researchers declined to comment on their experiment, which is being prepared for publication, but other experts praised the achievement.

“In a lot of my work, I’ve been doodling similar-looking pictures,” Ellison said. “It’s amazing to see that they actually demonstrated this.”

Paint by Measurement

All the while, a group of theorists headed up by Ashvin Vishwanath at Harvard University was quietly pursuing what many consider an even loftier goal: creating a more complicated phase of quantum matter where true non-abelian anyons — as opposed to defects — arise natively in a pristine phase of matter. “[Google’s] defect is kind of a baby non-abelian thing,” said Burnell, who was not involved in either effort.

Anyons of both types live in phases of matter with a topological nature defined by intricate tapestries of gossamer threads, quantum connections known as entanglement. Entangled particles behave in a coordinated way, and when trillions of particles become entangled, they can ripple in complicated phases sometimes likened to dances. In phases with topological order, entanglement organizes particles into loops of aligned spins. When a loop is cut, each end is an anyon.

Topological order comes in two flavors. Simple phases such as the toric code have “abelian order.” There, loose ends are abelian anyons. But researchers seeking true non-abelian anyons have their sights set on a completely different and much more complicated tapestry with non-abelian order.

Vishwanath’s group helped cook up a phase with abelian order in 2021. They dreamt of going further, but stitching qubits into non-abelian entanglement patterns proved too intricate for today’s unstable processors. So the crew scoured the literature for fresh ideas.

They found a clue in a pair of papers from decades before. Most quantum devices compute by massaging their qubits much as one might fluff a pillow, in a gentle way where no stuffing flies out through the seams. Carefully knitting entanglement through these “unitary” operations takes time. But in the early 2000s Robert Raussendorf, a physicist now at the University of British Columbia, hit on a shortcut. The secret was to hack away chunks of the wave function using measurement — the process that normally kills quantum states.

“It’s a really violent operation,” said Ruben Verresen, one of Vishwanath’s collaborators at Harvard.

Raussendorf and his collaborators detailed how selective measurements on certain qubits could take an unentangled state and intentionally put it into an entangled state, a process Verresen likens to cutting away marble to sculpt a statue.

The technique had a dark side that initially doomed researchers’ attempts to make non-abelian phases: Measurement produces random outcomes. When the theorists targeted a particular phase, measurements left non-abelian anyons speckled randomly about, as if the researchers were trying to paint the Mona Lisa by splattering paint onto a canvas. “It seemed like a complete headache,” Verresen said.

Toward the end of 2021, Vishwanath’s group hit on a solution: sculpting the wave function of a qubit grid with multiple rounds of measurement. With the first round, they turned a boring phase of matter into a simple abelian phase. Then they fed that phase forward into a second round of measurements, further chiseling it into a more complicated phase. By playing this game of topological cat’s cradle, they realized they could address randomness while moving step by step, climbing a ladder of increasingly complicated phases to reach a phase with non-abelian order.

“Instead of randomly trying measurements and seeing what you get, you want to hop across the landscape of phases of matter,” Verresen said. It’s a topological landscape that theorists have only recently begun to understand.

Last summer, the group put their theory to the test on Quantinuum’s H_1 trapped-ion processor, one of the only quantum devices that can perform measurements on the fly.

Replicating parts of Google’s experiment, they made the abelian toric code and created a stationary non-abelian defect in it. They tried for a non-abelian phase but couldn’t get there with only 20 qubits.

But then a researcher at Quantinuum, Henrik Dreyer, took Verresen aside. After swearing him to secrecy with a nondisclosure agreement, he told Verresen that the company had a second-generation device. Crucially, the H_2 had a whopping 32 qubits. It took substantial finagling, but the team managed to set up the simplest non-abelian phase on 27 of those qubits. “If we had one or two fewer qubits, I don’t think we could have done it,” Vishwanath said.

Their experiments marked the first unassailable detection of a non-abelian phase of matter. “To realize a non-abelian topological order is something people have wanted to do for a long time,” Burnell said. “That’s definitely an important landmark.”

Their work culminated in the braiding of three pairs of non-abelian anyons such that their trajectories through space and time formed a pattern known as Borromean rings, the first braiding of non-abelian anyons. Three Borromean rings are inseparable when together, but if you cut one the other two will fall apart.

“There’s a kind of gee-whiz factor,” Wilczek said. “It takes enormous control of the quantum world to produce these quantum objects.”

The Big Chill

As other physicists celebrate these milestones, they also emphasize that Google and Quantinuum are running a different race than the likes of Microsoft and Willett. Creating topological phases on a quantum processor is like making the world’s tiniest ice cube by stacking a few dozen water molecules — impressive, they say, but not nearly as satisfying as watching a slab of ice form naturally.

“The underlying math is extremely beautiful, and being able to validate that is definitely worthwhile,” said Chetan Nayak, a researcher at Microsoft who has done pioneering work on non-abelian systems. But for his part, he said, he’s still hoping to see a system settle into a state with this sort of intricate entanglement pattern on its own when cooled.

“If this was unambiguously seen in [Willett’s experiments], our minds would be blown,” Barkeshli said. Seeing it in a quantum processor “is cool, but no one’s getting blown away.”

The most exciting aspect of these experiments, according to Barkeshli, is their significance for quantum computation: Researchers have finally shown that they can make the necessary ingredients, 26 years after Kitaev’s initial proposal. Now they just need to figure out how to really put them to work.

One snag is that like Pokémon, anyons come in a tremendous number of different species, each with its own strengths and weaknesses. Some, for example, have richer memories of their pasts, making their braids more computationally powerful. But coaxing them into existence is harder. Any specific scheme will have to weigh such trade-offs, many of which aren’t yet understood.

“Now that we have the ability to make different kinds of topological order, these things become real, and you can talk about these trade-offs in more concrete terms,” Vishwanath said.

The next milestone will be real error correction, which neither Google nor Quantinuum attempted. Their braided qubits were hidden but not protected, which would have required measuring the crummy underlying qubits and quickly fixing their errors in real time. That demonstration would be a watershed moment in quantum computation, but it’s years away — if it’s even possible.

Until then, optimists hope these recent experiments will launch a cycle where more advanced quantum computers lead to a better command over non-abelian quasiparticles, and that control in turn helps physicists develop more capable quantum devices.

“Just bringing out the power of measurement,” Wilczek said, “that’s something that might be a game-changer.”

Correction: May 11, 2023

In Quantinuum’s early set of experiments, performed last summer, the team made a stationary non-abelian defect and confirmed it was non-abelian by braiding abelian anyons around it. They did not braid non-abelian defects, as the article originally stated.

Why Mathematicians Re-Prove What They Already Know

为什么数学家要重新证明他们已经知道的东西

几千年来，人们都知道质数是无限的，但新的证明让人们对定理之间的相互依赖有了新的认识。

The first proof that many people ever learn, early in high school, is the ancient Greek mathematician Euclid’s proof that there are infinitely many prime numbers. It takes just a few lines and uses no concepts more complicated than integers and multiplication.

His proof relies on the fact that, if there were a finite number of primes, multiplying them all together and adding 1 would imply the existence of another prime number. This contradiction implies that the primes must be infinite.

Mathematicians have a curiously popular pastime: proving it over and over again.

Why bother to do this? For one thing, it’s fun. More importantly, “I think the line between recreational math and serious math is very thin,” said William Gasarch, a professor of computer science at the University of Maryland and author of a new proof posted online earlier this year.

Gasarch’s proof is only the latest in a long succession of novel proofs. In 2018, Romeo Meštrović of the University of Montenegro compiled nearly 200 proofs of Euclid’s theorem in a comprehensive historical survey. Indeed, the whole field of analytic number theory, which uses continuously varying quantities to study the integers, arguably originated in 1737, when the mathematical giant Leonhard Euler used the fact that the infinite series 1 + 1/2 + 1/3 + 1/4 + 1/5 + … diverges (meaning that it doesn’t sum to a finite number), to again prove that there are an infinite number of primes.

Christian Elsholtz, a mathematician at Graz University of Technology in Austria and author of another recent proof, said that instead of proving hard results from many smaller results — what mathematicians do when they systematically assemble lemmas into theorems — he did the opposite. “I use Fermat’s Last Theorem, which is really a nontrivial result. And then I conclude a very simple result.” Working backward like this can reveal hidden connections between different areas of math, he said.

“There’s a little competition out there for people to have the most ridiculously difficult proof,” said Andrew Granville, a mathematician at the University of Montreal and author of two other proofs. “It has to be amusing. Doing something technically awful is not the point. The only way you want to do something difficult is that it’s amusing.”

Granville said that there is a serious point to this friendly one-upmanship. Researchers aren’t just fed questions that they try to solve. “The creation process in mathematics is not about, you just set a task to a machine and the machine resolves it. It’s about somebody taking what they’ve done in the past and using that to create a technique and create a way to develop ideas.”

As Gasarch puts it, “All the papers, they segue from a cute new proof that primes are infinite into serious math. One day you’re just looking at primes, and the next day you are looking at densities of squares.”

Gasarch’s proof begins with the fact that if you color the integers with a finite number of colors, there will always be a pair of numbers with the same color whose sum is also that color, which was proved in 1916 by Issai Schur. Gasarch used Schur’s theorem to show that, if there were a finite number of primes, then there would exist a perfect cube (an integer, like 125, that is equal to some other integer multiplied by itself three times) that is the sum of two other perfect cubes. But back in 1770, Euler had proved no such cube exists — the $n = 3$ case of Fermat’s Last Theorem, which posits that there are no integer solutions to $a^n + b^n = c^n$ for n larger than 2. Based on that contradiction, Gasarch reasoned that there must be an infinite number of primes.

One of Granville’s 2017 proofs used a different theorem of Fermat’s. Granville mainly relied on a 1927 theorem by Bartel Leendert van der Waerden, which showed that if you color the integers with a finite number of colors, there always exist arbitrarily long chains of evenly spaced integers with the same color. Like Gasarch, Granville started with the assumption that primes are finite. He then used van der Waerden’s theorem to find a sequence of four evenly spaced, identically colored perfect squares. But Fermat had proved that no such sequence can exist. Contradiction! Since such a sequence could exist were there a finite number of primes, but it can’t exist, there must be an infinite number of primes. Granville’s proof was the second recent prime proof to draw on van der Waerden’s theorem — Levent Alpöge, now a postdoc at Harvard University, had also used the result in a 2015 paper, published while he was still in college.

Granville is a particular fan of Elsholtz’s paper, which also applies Fermat’s Last Theorem and the counterfactual assumption that there are only finitely many primes. Like Gasarch, Elsholtz incorporated Schur’s theorem, though in a somewhat different way. Elsholtz also gave a second proof using a 1953 theorem by Klaus Roth, which says that sets of integers over a certain size must contain groups of three evenly spaced numbers.

Some deeper — and even practical — mathematical questions might be answered by building on this work. For example, public key encryption that relies on the difficulty of factoring large numbers would be very easy to break if we lived in a world with finitely many primes. Elsholtz wonders if there might therefore be some connection between the proofs of infinitely many primes and proving how hard it is to crack such encryption schemes. There is “some weak connection to Euclid’s theorem,” Elsholtz said. “It would be interesting to see the deeper connections.”

Granville said the best math can grow from strange combinations of different areas and subjects and often emerges after mathematicians have spent years noodling over lower-level but amusing problems. He is fascinated by the fact that seemingly remote subjects could be applied to number theory. In a recent survey, Granville praised the “sparse elegance” of a 1955 proof by Hillel Furstenberg, which used point-set topology. Like Alpöge, Furstenberg was still in college when his proof was published. He would go on to an illustrious career in a variety of mathematical disciplines.

Granville rhetorically asked if new proofs of Euclid’s old result are “just curiosity or something that has some long-term importance.” Answering his own question, he said: “I can’t tell you.”

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