华中科技大学数学中心

Center for Mathematical Sciences

中国 武汉

Wuhan China

Newsletter of the Center for Mathematical Sciences

2022 Autumn

华中科技大学数学中心 简介

在建设世界一流大学的征程中，数学学科的作用异常重要。华中科技大学高瞻远瞩，于2013年成立数学中心。华中科技大学数学中心一方面倡导数学不同分支之间的相互交叉，激发新的合作研究，催生新的研究领域和研究群体。另一方面引领数学与工科、理科，医科及其它学科之间的合作研究，实现交叉创新、合作共赢。

作为我校国际交流与合作的平台，数学中心大力推动与发展 “跨学科应用数学” 合作研究。我们的跨学科合作研究领域包括数学与地球科学（物理海洋学和气候动力学）的交叉研究，以及数学与生命科学（计算和定量生物学）的交叉研究。

华中科技大学数学中心积极开展前瞻性研究，立足华中、辐射全国、影响海外。数学中心将国际先进的人才培养模式和研究机构运行机制有机融入到我国建设一流大学与一流学科的伟大事业之中，努力成为培养和聚集一流人才的平台，国际交流与合作的平台，科教运行机制以及人事体制改革试点的平台。

数学中心成员包括院士，国家特聘专家，外专千人计划专家，长江学者，青年学术英才，楚天学者，洪堡学者和华中学者。还有一批海内外知名访问学者，博士后，博士生，以及来自多个国家的留学生。数学中心设有李国平讲座教授，东湖讲座教授，东湖数学论坛，和郭友中数理科学讲座。

希望重要的数学发现萌芽于此，
希望新的研究领域和研究群体产生于此，
希望著名数学家和科学家在此留下足迹，
希望科技界更深刻地感受到数学的作用:
数学强，则科技强；科技强，则国家强！

数学中心官网

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華中科技大學 数学中心
Center for Mathematical Sciences

Web:mathcenter. hust. edu. cn   E-mail：mathcenter@hust.edu.cn

华中科技大学数学中心招收2023年免推硕士研究生

华中科技大学数学中心招收2023年秋季入学免推硕士研究生（面向有免推资格的本科生。本科专业不限）。

招生视频：http://x.eqxiu.com/s/3obeLrBg?eqrcode=1

华中科技大学数学中心网站：http://mathcenter.hust.edu.cn/

研究领域：包括随机动力系统，随机偏微分方程，随机分析，动力系统及其应用，几何与拓扑，偏微分方程，计算数学，应用数学，图像科学，数据科学与统计学，多尺度系统建模与计算模拟，数理地球科学和定量生物学。研究生指导团队实行双导师制，由本校专家和海外学者组成，包括院士，国家特聘专家，长江学者，青年学术英才，东湖讲座教授，楚天学者，优青，洪堡学者和华中学者。

数学中心已有来自世界各国的优秀学者，包括院士，教授，副教授/副研究员，助理教授，客座教授、访问学者，博士后以及博士研究生。他们从国内外（包括美国、英国、法国、德国、澳大利亚等国家）汇聚到美丽的江城武汉东湖之滨，共同致力于基础数学，计算与应用数学，概率与统计，数据科学，数学物理与交叉科学的发展。

欢迎有意愿的学生联系华中科技大学数学中心段金桥主任

（电邮：mathcenter@hust.edu.cn.）

欢迎加盟华中科技大学数学中心！

数学中心网址

招生视频

華中科技大學 数学中心

Center for Mathematical Sciences

Wuhan, China

Web：mathcenter. hust. edu. cn　　Email: mathcenter@hust.edu.cn

数学正在发生日新月异的变化。不仅数学内部各分支相互交融，共同推动数学向更高层次发展，而且科学与工程问题牵涉到越来越深的数学课题，对数学提出了重大挑战，激发了新的数学理论和方法的创立，从而推动数学本身的发展。数学也一直在背后推动着科学和工程技术的进步，为现代科学和高新技术的发展奠定坚实基础。世界强国必须是数学强国，数学弱国不可能是现代化强国，而现代高科技竞争同时包含数学研究的竞争。华中科技大学数学中心顺应科学发展趋势于2013年在武汉成立了。

数学中心宗旨

（1） 积极倡导数学不同分支之间的交叉研究；激发新的合作探索，催生新的研究领域和研究群体；

（2） 努力推动数学与科学，工程，医学之间的交叉研究；建立数学家和科学家之间的广泛联系，从而达到合作共赢；

（3） 聚集一流人才，培养优秀学生，做出一流学术研究，引领学科发展，服务国家和社会。

数学中心成员

数学中心已有来自世界各国的优秀学者，包括院士，教授，副教授/副研究员，助理教授，客座教授、访问学者，博士后以及博士研究生。他们从国内外（包括美国、英国、法国、德国、澳大利亚等国家）汇聚到美丽的江城武汉东湖之滨，共同致力于基础数学，计算与应用数学，概率与统计，数据科学，数学物理与交叉科学的发展。

数学中心诚聘海内外英才

无论您来自哪里，数学是我们的共同语言，欢迎您加入我们!

数学中心招聘网址：
https://www.mathjobs.org/jobs/list/16913

申请材料请寄段金桥老师:
mathcenter@hust.edu.cn

数学中心招聘二维码：

Open Positions in Wuhan Math Center

The Center for Mathematical Sciences, at Huazhong University of Science and Technology, Wuhan, China, is a mathematical research and education institution. Our mission is to promote interactions between mathematics and other disciplines, and to connect branches of mathematics. The Center's current research themes include stochastic mathematics, applied/computational mathematics, core mathematics, mathematical physics, and data science. Distinguished applicants in all areas of mathematical sciences will be considered.

Open Positions

• Assistant/Associate Professorships

Basic requirements:
A PhD degree in mathematical sciences or related fields, with distinguished research credentials.

• Postdoctoral fellows

Basic requirements:
New or recent PhD graduates in mathematical sciences or related fields, with distinguished research potential.

The compensation packages, including salary, start-up funds and housing allowance, are highly competitive, and are commensurate with qualification and experience.
The Center will also sponsor qualified candidates to compete for the National Junior Endowed Professorships.

How to Apply: Applicants should send these materials to mathcenter@hust.edu.cn:

1） A cover letter;
2） A curriculum vita with a list of 3 references;
3） A research statement.

Contact Us

Email Professor Jinqiao Duan: mathcenter@hust.edu.cn
Web Page: mathcenter.hust.edu.cn
Ads in MathJobs *https://www.mathjobs.org/jobs/list/16913*

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News
新闻

第一届华中南代数拓扑几何研讨会

华中科技大学数学中心&华南理工大学数学学院于 2022 年 7 月 18 日至 2022 年 7 月 21 日顺利举办第一届华中南代数拓扑几何研讨会。

学术交流

短课

1. 2022 年 9 月德国哥廷根大学朱晨畅教授来访并在数学中心讲授《Higher symplectic stacks in differential geometry》短课。10 月在数学与统计学院做《浅谈（高阶）范畴》的报告。

短课主题：Higher symplectic stacks in differential geometry
授课老师：朱晨畅 教授（德国哥廷根大学）
时间：2022 年 10 月 6 日-10 月 9 日
Zoom ID: 889 1241 3953　　Passcode: 330378

课程安排：

Schedule	Outline
2022 年 10 月 6 日 3:00-3:50 pm: Lecture 4:00-4:30 pm: Discussion	● Kan simplicial objects with Grothendieck pretopologies, ● Lie n-groupoids;
2022 年 10 月 7 日 3:00-3:50 pm: Lecture 4:00-4:30 pm: Discussion	● Lie 2-groups as categorification of Lie groups, ● Morita equivalence: via a) hypercovers, b) weak equivalences, c) bibundles (sometimes called Hilsum-Skandalis bibundles);
2022 年 10 月 8 日 3:00-3:50 pm: Lecture 4:00-4:30 pm: Discussion	● NQ manifolds (a sort of d.g. manifolds), ● Lie n-algebroids (as tangent complex of Lie n-groupoids);
2022 年 10 月 9 日 3:00-3:50 pm: Lecture 4:00-4:30 pm: Discussion	● m-shifted symplectic Lie n-groupoids, with example of BG (or Lie group) together with several interesting models of symplectic forms, symplectic Morita equivalence, I.M. (infinitesimal multiplicative) forms on Lie n-algebroids, which provide models of symplectic forms.

- 报告人简介：朱晨畅，德国哥廷根大学终身教授，奥林匹克数学竞赛金牌（满分）得主。1999 年在北京大学获得学士学位，2004 年在加州大学伯克利分校获得博士学位，瑞士苏黎世联邦理工学院博士后，2013 年在德国哥廷根大学获得终身职位。从事 Poisson 几何，李群胚等高阶微分几何的研究。在 Duke Math. J., Compos. Math., Adv. Math., JEMS, Math. Ann., Trans. Amer. Math. Soc., Comm. Math. Phys., IMRN, Ann. Inst. Fourier (Grenoble), 等杂志上发表高水平论文 30 余篇。

2. 2022 年 7-8 月美国莱特州立大学王维真教授在数学中心讲授《精确统计推断选讲及其在生物统计中的应用》暑期短课。

课程名：精确统计推断选讲及其在生物统计中的应用
课程教师：王维真，Professor Weizhen Wang
课程时间：7 月：25，27，29；8 月：1，3，5，8，10；晚：6:30-9:30 pm
https://meeting.tencent.com/dm/lH4JcdI2Z7uX
腾讯会议：537-5597-9896　密码：123456
课程摘要：

- 介绍精确统计推断的概念，特点，和使用它的必要性。讨论近似推断的局限性以及改进其成为精确推断的一个一般性方法。描述精确推断在多种参数估计中的应用，和它们在生物统计，特别是临床试验中的应用。内容涉及当前最新统计学结果和未解课题。

学术报告

报告题目：Online algorithms for data representation

报告人：李乐（华中师范大学）

时间：2022.08.26，下午 14:00-15:00（北京时间）

地点：华中科技大学恩明楼数学中心 813

报告摘要：

- In the most basic version of online learning, the forecaster gets access one after another to a sequence $z_1, z_2, \ldots$ of elements. At each time t=1, 2, …, before z_t is revealed, the forecaster gives his guess of value of z_t on the basis of the previous observations and other available side information. We considered the problem of online clustering and sequential learning of principal curves. For the former, we introduced a new and adaptive online clustering algorithm relying on a quasi-Bayesian approach, with a dynamic estimation of the number of clusters. We proved both regret bounds for the algorithm and gave a corresponding RJMCMC-flavored implementation. For the latter, inspired also by the quasi-Bayesian idea, we gave an algorithm relying on the mode of Gibbs-posterior and proved the sublinear regret bound for it.

更多课程回放内容，关注哔哩哔哩官方账号：华中大数学中心

国际会议

- The 7th International Conference on Random Dynamical Systems, Hanoi,21-25, June.
  作报告的有：魏崴，张奥，胡建宇，高婷，袁胜兰，卢裕斌，黄乔，段金桥。

- 数学中心师生 2022 年 6 月 27 日-7 月 1 日参加第 42 届 SPA 会议（随机过程及其应用 Stochastic Processes and their Applications），第 42 届 SPA 会议由国际伯努利学会主办，是关于随机过程理论及其应用的最重要的大型国际学术会议，武汉大学承办，在武汉东湖国际会议中心举行。

- 数学中心师生参加 7 月 30 日-8 月 5 日第九届世界华人数学家大会（ICCM），大会在南京举办。

Academic Achievement

学术成果

数学中心 7—9 月已发表文章

1. Extracting stochastic governing laws by non-local Kramers–Moyal formulae

——Yubin Lu, Yang Li, Jinqiao Duan

Philosophical Transactions of the Royal Society A

https://doi.org/10.1098/rsta.2021.0195

➢ With the rapid development of computational techniques and scientific tools, great progress of data-driven analysis has been made to extract governing laws of dynamical systems from data. Despite the wide occurrences of non-Gaussian fluctuations, the effective data-driven methods to identify stochastic differential equations with non-Gaussian Lévy noise are relatively few so far. In this work, we propose a data-driven approach to extract stochastic governing laws with both (Gaussian) Brownian motion and (non-Gaussian) Lévy motion, from short bursts of simulation data. Specifically, we use the normalizing flows technology to estimate the transition probability density function (solution of non-local Fokker–Planck equations) from data, and then substitute it into the recently proposed non-local Kramers–Moyal formulae to approximate Lévy jump measure, drift coefficient and diffusion coefficient. We demonstrate that this approach can learn the stochastic differential equation with Lévy motion. We present examples with one- and two-dimensional decoupled and coupled systems to illustrate our method. This approach will become an effective tool for discovering stochastic governing laws and understanding complex dynamical behaviours.

2. Analysis of multiscale methods for stochastic dynamical systems driven by α-stable processes

——Yanjie Zhang, Xiao Wang, Zibo Wang, Jinqiao Duan

Applied Mathematics Letters

https://doi.org/10.1016/j.aml.2022.108462

➢ In this paper, we firstly analyze the strong convergence of projective integration method for multiscale stochastic dynamical systems driven by α-stable processes, which is used to estimate the effect that the fast components have on slow ones. Then we obtain the p th moment error bounds between the solution of slow component produced by projective integration method and the solution of effective system with $p \in (1, \alpha)$. Finally, we corroborate our analytical results through a specific numerical example.

3. Homogenization of nonlocal partial differential equations related to stochastic differential equations with Lévy noise

——Qiao Huang, Jinqiao Duan, Renming Song

Bernoulli

https://doi.org/10.3150/21-BEJ1365

➢ We study the "periodic homogenization" for a class of nonlocal partial differential equations of parabolic-type with rapidly oscillating coefficients, related to stochastic differential equations driven by multiplicative isotropic α-stable Lévy noise ($1<\alpha<2$) which is nonlinear in the noise component. Our homogenization method is probabilistic. It turns out that, under suitable regularity assumptions, the limit of the solutions satisfies a nonlocal partial differential equation with constant coefficients, which are associated to a symmetric α-stable Lévy process.

4. Bursting hierarchy in an adaptive exponential integrate-and-fire network synchronization

——Congping Lin, Xiaoyue Wu, Yiwei Zhang

Biological Cybernetics

https://doi.org/10.1007/s00422-022-00942-9

➢ Neuronal network synchronization has received wide interest. In the present manuscript, we study the influence of initial membrane potentials together with network topology on bursting synchronization, in particular the sequential order of stabilized bursting among neurons. We find a hierarchical phenomenon on their bursting order. With a focus on situations where network coupling advances spiking times of neurons, we grade neurons into different layers. Together with the neuronal network structure, we construct directed graphs to indicate bursting propagation between different layers. More explicitly, neurons in upper layers burst earlier than those in lower layers. More interestingly, we find that among the same layer, bursting order of neurons is mainly associated with the number of neurons they connected to the upper layer; more stimuli lead to earlier bursting. Receiving effectively the same stimuli from the upper layer, we observe neurons with fewer connections would burst earlier.

5. Hochschild cohomology of dg manifolds associated to integrable distributions

——Zhuo Chen, Maosong Xiang, Ping Xu

Communications in Mathematical Physics
https://doi.org/10.1007/s00220-022-04473-z

- For the field $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$, and an integrable distribution $F \subseteq T_M \otimes_{\mathbb{R}} \mathbb{K}$ on a smooth manifold *M*, we study the Hochschild cohomology of the dg manifold $(F[1], d_F)$ and establish a canonical isomorphism with the Hochschild cohomology of the algebra of functions on leaf space in terms of transversal polydifferential operators of *F*. In particular, for the dg manifold $(T_X^{0,1}[1], \bar{\partial})$ associated with a complex manifold *X*, we prove that its Hochschild cohomology is canonically isomorphic to the Hochschild cohomology $HH^{\bullet}(X)$ of the complex manifold *X*. As an application, we show that the Duflo-Kontsevich type theorem for the dg manifold $(T_X^{0,1}[1], \bar{\partial})$ implies the Duflo-Kontsevich theorem for complex manifolds.

Qualifying Exams
资格考试

Probability Theory Qualifying Exam

Center for Mathematical Sciences Huazhong University of Science and Technology

mathcenter.hust.edu.cn

Fall 2022

Note: This exam covers the book "Probability-Second Edition", by A.N.Shiryaev.

Total: 100 points (10 points for each problem).

1. (10 points) Let the random variables $\eta_1,\ldots,\eta_k$ satisfy $E(\eta_k|\eta_1,\ldots,\eta_{k-1})=0$. Show that the sequence $\xi=(\xi_k)_{1\le k\le n}$ with $\xi_1=\eta_1$ and

$$\xi_{k+1}=\sum_{i=1}^{k}\eta_{i+1}f_i(\eta_1,\ldots,\eta_i),$$

where f_i are given functions, is a martingale.

2. (10 points) Prove that:
   (a) $\overline{\liminf A_n}=\limsup \overline{A}_n$
   (b) $\limsup A_n \cap \liminf B_n \subseteq \limsup(A_n\cap B_n)\subseteq \limsup A_n \cap \limsup B_n$.
   (c) If $A_n\uparrow A$ or $A_n\downarrow A$, then $\liminf A_n=\limsup A_n$.

3. (10 points) Let $(\Omega,\mathcal{F},\mathrm{P})$ be a probability space and $\mathcal{A}$ an algebra of subsets of Ω such that $\sigma(\mathcal{A})=\mathcal{F}$. Using the principle of appropriate sets, prove that for every $\varepsilon>0$ and $B\in\mathcal{F}$ there is a set $A\in\mathcal{A}$ such that
$$P(A\Delta B)\le\varepsilon.$$

4. (10 points) Let ξ and η be random variables on $(\Omega,\mathcal{F})$, and $A\in\mathcal{F}$. Then the function
$$\xi(\omega)=\xi(\omega)\cdot I_A+\eta(\omega)I_{\overline{A}}$$
is also a random variable.

5. (10 points) Let $(\xi_n)_{n\ge1}$ have the property that $\sum_{n=1}^{\infty}E|\xi_n|^p<\infty$ for some $p>0$. Show that $\xi_n\to0$ (P-a.s.).

6. (10 points)
   (a) Write the definition of the conditional expectation of a negative random variable ξ with respect to the σ-algebra $\mathcal{G}$.
   (b) Let $\xi_1,\xi_2,\ldots$ be independent identically distributed random variables with

$E|\xi_i|<\infty$. Show that

$$E(\xi_1|S_n,S_{n+1},\ldots)=\frac{S_n}{n} \quad \text{(a.s.)},$$

where $S_n=\xi_1+\ldots+\xi_n$.

7. (10 points) Using Fatou's lemma, show that

$$\mathrm{P}(\lim A_n)\le \underline{\lim}\lim \mathrm{P}(A_n), \qquad \mathrm{P}(\overline{\lim}A_n)\ge \overline{\lim}\lim \mathrm{P}(A_n)$$

8. (10 points) Suppose that the random elements (X,Y) are such that there is a regular distribution $P_x(B)=\mathrm{P}(Y\in B|X=x)$. Show that if $E|g(X,Y)|<\infty$ then

$$E\left[g(X,Y)|X=x\right]=\int g(x,y)P_x(dy) \qquad (P_x\text{-a.s.})$$

9. (10 points)
(a) Write the Borel-Cantelli Lemma.
(b) Prove the Borel-Cantelli Lemma.

10. (10 points) Let $(\xi_n)_{n\ge1}$ be a sequence of independent identically distributed random variables. Show that

$$E|\xi_1|<\infty \Leftrightarrow \sum_{n=1}^{\infty}P\{|\xi_1|>\varepsilon\cdot n\}<\infty$$

$$\Leftrightarrow \sum_{n=1}^{\infty}P\left\{\left|\frac{\xi_n}{n}\right|>\varepsilon\right\}<\infty \Rightarrow \frac{\xi_n}{n}\to 0 \quad \text{(P-a.s.)}.$$

Exam for Numerical Analysis

Center for Mathematical Sciences Huazhong University of Science and Technology

Note: This exam covers J.Stoer & R.Bulirsch: Introduction to Numerical Analysis Chapters 3-7. Total 100 points (20 points for each problem).

1. If $f \in C^2[a,b]$ then there exists an $\tilde{x} \in (a,b)$ such that the error of the trapezoidal rule is expressed as follows:
$$\frac{1}{2}(b-a)(f(a)+f(b)) - \int_a^b f(x)dx = \frac{1}{12}(b-a)^3 f''(\tilde{x}).$$
Derive this result from the error formula (which is in (2.1.4.1)) by showing that $f''(\xi(x))$ is continuous in x.

2. Let $Ax = b$ be given with
$$A = \begin{bmatrix} 0.780 & 0.563 \\ 0.913 & 0.659 \end{bmatrix} \text{ and } b = \begin{bmatrix} 0.217 \\ 0.254 \end{bmatrix}.$$
The exact solution is $x^T = (1,-1)$. Further, let two approximate solutions
$$x_1^T = (0.999, -1.001) x_2^T = (0.341, -0.087)$$
be given.
(a) Compute the residuals $r(x_1)$, $r(x_2)$. Does the more accurate solution have a smaller residual?
(b) Determine the exact inverse A^{-1} of A and cond(A) with respect to the maximum norm.
(c) Express $\tilde{x} - x = \Delta x$ using $r(\tilde{x})$, the residual for $\tilde{x}$.

3. Give an iterative method for computing $\sqrt[n]{a}$, $a>0$, which converges locally in second order. (The method may only use the four fundamental arithmetic operations.)

4. For $u, \upsilon, \omega, z \in \mathbb{R}^n$, $n>2$, let
$$A := u\upsilon^T + \omega z^T.$$
(a) With λ_1, λ_2 the eigenvalues of
$$\tilde{A} := \begin{bmatrix} \upsilon^T u & \upsilon^T \omega \\ z^T u & z^T \omega \end{bmatrix},$$
show that A has the eigenvalues $\lambda_1, \lambda_2, 0$.
(b) How many eigenvectors and principal vectors can A have? What types of Jordan normal form J of A are possible? Determine J in particular for $\tilde{A} = 0$.

5. Let $\eta(x;h)$ be the approximate solution furnished by Euler's method for the initial-value problem
$$y' = y, \ y(0) = 1.$$
(a) One has $\eta(x;h) = (1+h)^{x/h}$.
(b) Show that $\eta(x;h)$ has the expansion

$$\eta(x;h)=\sum_{i=0}^{\infty}\mathcal{T}_i(x)h^i \quad \text{with} \quad \mathcal{T}_0(x)=e^x,$$

which converges for $|h|<1$; the $\mathcal{T}_i(x)$ here are analytic functions independent of h.

(c) Determine $\mathcal{T}_i(x)$ for $i=1,2,3$.

(d) The $\mathcal{T}_i(x),\ i\ge 1$, are the solutions of the initial-value problems

$$\mathcal{T}_i'(x)=\mathcal{T}_i(x)-\sum_{k=1}^{i}\frac{\mathcal{T}_{i-k}^{k+1}(x)}{(k+1)!},\ \mathcal{T}_i(0)=0.$$

Popular Mathematics
数学热门话题

e is everywhere
e 无处不在

e 在人类科学中有极为广泛的应用，在热力学、统计学、信息论、通信工程、甚至经济学社会学当中，到处都能发现它的身影。

From determining the compound interest on borrowed money to gauging chances at the roulette wheel in Monte Carlo, Stefanie Reichert explains that there's no way around Euler's number.

Even outside school or university, we cannot escape Euler's number. Jacob Bernoulli is credited with discovering e while thinking about matters of continuous compound interest in 1683. He realized that when the compounding period became smaller and smaller and more and more periods were considered, the amount of money would converge towards a limit that was later found to be one of the representations of e. Since then, use of Euler's number has become more widespread and now it appears in many branches of science and in everyday life.

For example, Euler's number shows up in probability theory. Imagine you are in Monte Carlo enjoying a few games of roulette, which is a Bernoulli trial process. If you place a bet on a single number, your chances are 1/37 to win that game. For 37 games, the probability that you will lose every single time is — maybe surprisingly — close to 1/e. Or, pretend you are at the theatre, where you — along with everybody else — leave your coat in the cloak room, which has one hook per guest, and receive a number. However, your coat is placed on a random hook. The probability that none of the coats are on the correct hook for a large number of guests approaches, again, 1/e. The number of practical examples is endless.

The history of e reads like the Who's Who of mathematics and physics. It all started with the discovery of the logarithm by John Napier: Euler's number is hidden deep in the many pages of the appendix tabulating natural logarithms in his 1614 work *Mirifici Logarithmorum Canonis Descriptio*. Later, when Bernoulli studied the case of continuous compound interest, he concluded that the limit must converge to a number between 2 and 3. As it turned out, this limit equals Euler's number (less commonly known as Napier's constant), and Bernoulli came up with its first approximation.

It took a while before scholars connected the dots and realized that the base of the logarithm introduced by Napier and the limit discovered by Bernoulli were closely related and settled on a common notation. Gottfried Leibniz referred to what is now known as Euler's number as b in discussions with Christiaan Huygens, whereas others such as Jean-Baptiste le Rond d'Alembert preferred to use the notation c instead. This dispute was eventually settled when the Swiss mathematician Leonhard Euler (pictured) used the letter e in an early essay on the firing of cannons — and his choice became increasingly popular.

Similar to π, Euler's number e ≈ 2.71828 is irrational and also transcendental —meaning it doesn't form a solution of a non-zero polynomial equation with integer coefficients. Whether e (or π) is a normal number remains to be determined. A normal number consists of a sequence of digits in which single digits between 0 and 9 occur with a frequency of 10%, whereas each pair of digits between 00 and 99 occurs with a frequency of 1%, and so on.

Euler is credited with a whole bunch of constants besides e, so one should be careful not to mix Euler's number up with Euler's constant, also called the Euler–Mascheroni constant, γ ≈ 0.57721, defined as the limit of the difference between the harmonic series and the natural logarithm. The Euler–Mascheroni constant appears, for example, in the Bessel function of the second kind, and has not been proven to be irrational or transcendental. Another tricky case are Euler numbers (also known as zig or secant numbers), referring to the number of odd alternating permutations in expressions for the secant and hyperbolic secant (https://go.nature.com/2N0G3tc). To complicate things further, at least three other mathematical terminologies are in use denoting the Euler number of a finite complex, Euler primes or the Euler characteristics, a topological invariant. And in fluid dynamics, the Euler number characterizes the energy loss in a flow.

We have all encountered Euler's number in more ways than one — from natural logarithms to the definition of the exponential function, which relies on the series expansion of e discovered by Euler himself in 1748. The constant e appears practically everywhere in science: popping up in the definition of the standard normal distribution; allowing us to decompose a time-dependent signal into its frequencies via Fourier transformation; telling us how to calculate the half-life of radioactive elements; playing a crucial role in the growth of bacteria; and governing temperature-activated chemical reactions.

m e Å s u R E_{h} F Ω °R μ_0 ε_0 α σ V R e

Not for nothing, e counts among the most important constants in mathematics and physics, along with 0, 1, i and π that all show up in Euler's identity $e^{i\pi} + 1 = 0$. It is truly a constant in everyone's life.

Fields of joy
赞扬菲尔兹奖

菲尔兹奖（*Fields Medal*），又译为菲尔茨奖，是依加拿大数学家约翰•查尔斯•菲尔兹（*John Charles Fields*）要求设立的国际性数学奖项，于 1936 年首次颁发。菲尔兹奖是数学领域的国际最高奖项之一。因诺贝尔奖未设置数学奖，故该奖被誉为“数学界的诺贝尔奖”。

菲尔兹奖每四年颁发一次，在由国际数学联合会主办的四年一度的国际数学家大会上举行颁奖仪式，每次授予 2 至 4 名有卓越贡献的数学家。获奖者必须在该年元旦前未满 40 岁，每人能获得 1.5 万加拿大元奖金和金质奖章一枚。

截至 2018 年，世界上共有 60 位数学家获得菲尔兹奖，其中 2 位为华裔数学家，分别是 1982 年获奖的数学家丘成桐和 2006 年获奖的数学家陶哲轩。世界各高校按照最多获奖人数的排名依次为美国哈佛大学（18 位）、法国巴黎大学/十三所大学统称（16 位）、美国普林斯顿大学（15 位）、法国巴黎高等师范学院（14 位）、美国加利福尼亚大学伯克利分校（14 位）。

The announcement of the 2018 Fields Medal winners, made last month at the opening of the International Congress of Mathematicians held in Rio de Janeiro, Brazil, was greeted with widespread acclaim well beyond the mathematics community.

Of the four winners, the work of Alessio Figalli on optimal transport, which seeks the most efficient way to distribute goods on a network, is probably the most straightforward to connect to the real world. The recognized contributions of the other three awardees, Caucher Birkar, Peter Scholze and Akshay Venkatesh, are on more abstract topics concerning algebraic varieties, *p*-adic fields and number theory, respectively — humbling subjects even for the most theoretically inclined of physicists.

Although often referred to as the Nobel Prize of mathematics, the Fields Medal is in fact very different in terms of its procedures, criteria, remuneration and much else. Notably, the Nobel is typically given to senior figures, often decades after the contribution being honoured. By contrast, Fields medallists must all be under 40, an age at which, in most sciences, a promising career would just be taking off.

Indeed, the most famous instruction left by the prize’s main proponent, John Charles Fields, was that the awards should be both “in recognition of work already done” and “an encouragement for further achievement”.

So let us celebrate the work of these brilliant young mathematicians. And let us encourage them to further achievements that will doubtless spill into physics as well.

Mathematics, the queen of sciences
数学，科学的女王

我们强调了今年数学领域一些最重要奖项的获奖者如何对计算科学界产生影响。

On 5 July 2022, the International Mathematical Union (IMU), an international non-governmental and non-profit scientific organization, announced the awardees of some of the most prestigious and important prizes in mathematics. All of these awards represent important scientific contributions that substantially move the field of mathematics — and science as a whole — forward. Some of the awards from this year are also particularly noteworthy for the computational and computer science communities.

The Fields Medal, often described as the Nobel Prize of mathematics, recognizes outstanding mathematical achievements and is awarded to mathematicians under 40 years of age. This year, there were four medalists in total; among the awardees, Hugo Duminil-Copin, a professor at the Institute of Advanced Scientific Studies in France and at the University of Geneva in Switzerland, works on the intersection between mathematics and physics. One of his most notable contributions is the development of mathematical models that can be used to explore a fundamental physical phenomenon known as phase transition: as an example, when ice melts, a phase change occurs and solid transforms into liquid water. In their work, Duminil-Copin and colleagues extended the well-understood two-dimensional Ising model to higher dimensions; notably, this extension resolved the continuity problem of phase transition in three-dimensional Ising models.

The IMU Abacus Medal, previously known as the Rolf Nevanlinna Prize, is awarded to theoretical computer scientists under 40 years of age for outstanding contributions in the mathematical aspects of information sciences, including computer science, scientific computing and numerical analysis. Notable winners of the previous Rolf Nevanlinna Prize include, for instance, Leslie Valiant, for his many contributions to theoretical computer science; Peter Shor, for his work on quantum computation, and more specifically, for deriving Shor's algorithm; and Jon Kleinberg, for his contributions to the mathematical theory of the global information environment, including small-world networks and the theory underlying search engines. This year, the Abacus Medal goes to Mark Braverman, a professor of computer science at Princeton University, for bringing mathematical rigor — from information theory — into communication complexity, an area that considers scenarios where there are multiple parties performing computation. Braverman's work focuses on developing techniques for proving precise estimates on the amount of communication needed between multiple parties, with the goal of minimizing the amount of information that they need to share to complete their task: in other words, how can the task be accomplished with each party learning as little as possible from each other? Practically, this has implications on various real-world settings that depend on interactive communication, such as information security, data compression, and the design of streaming algorithms.

Another notable awardee for the computational science community is Elliott Lieb, a professor of mathematics and Higgins professor of physics at Princeton University, who is the recipient of this year's Carl Friedrich Gauss Prize. This prize is awarded for outstanding mathematical contributions that have found important applications outside of mathematics; in Lieb's case, his contributions have had extraordinary breadth, with implications in fields such as quantum mechanics, quantum information theory, and computational chemistry. Among many achievements, Lieb and colleagues solved the two-dimensional Ising model; proved the strong subadditivity of quantum entropy, which is a basic theorem in quantum information theory; and provided various proofs of thermodynamic functions, including in the homogenous electron gas, which serves as the basis of many functionals in density functional theory. Lieb's work on determining various inequalities has contributed to the calibration of density functionals, the understanding of the stability of matter, and the establishment of constants in functional analysis to assess nonlinear quantum systems.

It is certainly not surprising that the concepts and advances in the field of mathematics are widely important to other areas of research, including computational science. Carl Friedrich Gauss, the famous mathematician after which one of the prizes is named, is said to have stated that mathematics is 'the queen of sciences'. We could not agree more with such a statement.

Machine learning to guide mathematicians
指导数学家的机器学习

By Fernando Chirigati

我们通常认为，数学家的世界充满了直觉和想象力，他们发现模型、提出猜想、证明定理；而计算机只不过是擅长机械的计算。但能够从大量数据中学习的 AI，是否能够像数学家一样，从数据中发现模式？是否可以辅助数学家做出新发现呢？12 月 1 日，DeepMind 团队在 Nature 杂志上发表的一项最新研究中，人们成功让 AI 与人类数学家进行了合作，利用机器学习从大规模数据中探测模式，然后数学家尝试据此提出猜想，精确表述猜想并给出严格证明。他们解决了纯数学领域的两个问题：得到了纽结理论中代数和几何不变量之间的关系，提出了表示论中组合不变性猜想的可能证明方法。这次成功意味着未来机器学习可能会被引入数学家的工作中，AI 和数学家之间将展开更深入的合作。有数学家认为，这就像是伽利略有了望远镜，能够凝视数据宇宙的深处，看到之前从未探测到的东西。

Conjectures, which are propositions that are suspected to be true (often because of a consistent observed pattern) but that no one has been able to prove or refute, are incredibly important within the field of mathematics. Since the advent of computers, mathematicians have had a powerful technology at their disposal, which has helped to accelerate the investigation of conjectures. For instance, computational techniques such as machine learning (ML) have been used to directly and automatically generate conjectures. However, mathematicians must then go over the multiple auto-generated conjectures and try to further understand and prove them. What if ML could be used to guide the intuition of an expert mathematician, instead of taking the center stage of this process? In a recent work, Alex Davies, Pushmeet Kohli and colleagues proposed a framework to do exactly that: the mathematician has a hypothesis based on her expertise, and the framework helps her to verify and interpret such a hypothesis, guiding her in identifying conjecture candidates that may be worth pursuing.

The framework works as follows. The mathematician has a hypothesis that two mathematical objects, $X(z)$ and $Y(z)$, are related. The first step is to generate a dataset of $X(z)$ and $Y(z)$ pairs. Then, in the next step, supervised learning is used to train a function $\hat{f}$ that predicts $Y(z)$ using $X(z)$ as input: $\hat{f}$, in this case, represents the relationship between these objects, meaning, the hypothesis raised by the mathematician. A key advantage here is that, by using supervised learning, a broad set of nonlinear functions could be learned. If the accuracy obtained with $\hat{f}$ is statistically above chance, which indicates that the relationship might indeed be true, attribution techniques are used to better interpret the relationship. This makes it possible to find which features are more pertinent for the predictions of $Y(z)$, and therefore potentially more relevant to further investigate. This entire process, from refining the hypothesis and generating data to training the model and interpreting the results, can be iteratively repeated until the mathematician reaches a viable conjecture using the learned relationship and the most relevant features. Note that, in this work, the mathematician takes the center stage and drives the framework by using her expertise to refine the results as appropriate.

Notably, the authors used this framework in two distinct areas of mathematics (knot theory and combinatorial representation theory), identifying previously unknown relationships in these areas. But the potential implications of their approach go beyond these two areas: the proposed framework has the capability to put ML in the vanguard of mathematical research, but by guiding, and not replacing, the unique expertise of our mathematicians.

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