中国 武汉
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華中科技大學数学中心
Center for Mathematical Sciences

Newsletter, Spring 2024

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華中科技大學 数学中心
Center for Mathematical Sciences

华中科技大学数学中心简介

在建设世界一流大学的征程中，数学学科的作用异常重要。华中科技大学高瞻远瞩，于2013年成立数学中心。华中科技大学数学中心一方面倡导数学不同分支之间的相互交叉，激发新的合作研究，催生新的研究领域和研究群体。另一方面引领数学与工科、理科，医科及其它学科之间的合作研究，实现交叉创新、 合作共赢。

作为我校国际交流与合作的平台，数学中心大力推动与发展“跨学科应用数学”合作研究。我们的跨学科合作研究领域包括数学与地球科学（物理海洋学和气候动力学）的交叉研究，以及数学与生命科学（计算和定量生物学）的交叉研究。

华中科技大学数学中心积极开展前瞻性研究，立足华中、辐射全国、影响海外。数学中心将国际先进的人才培养模式和研究机构运行机制有机融入到我国建设一流大学与一流学科的伟大事业之中，努力成为培养和聚集一流人才的平台， 国际交流与合作的平台，科教运行机制以及人事体制改革试点的平台。

数学中心成员包括院士，国家特聘专家，外专千人计划专家，长江学者，青年学术英才，楚天学者，洪堡学者和华中学者。还有一批海内外知名访问学者，博士后，博士生，以及来自多个国家的留学生。数学中心设有李国平讲座教授，东湖讲座教授，东湖数学论坛，和郭友中数理科学讲座。

希望重要的数学发现萌芽于此，
希望新的研究领域和研究群体产生于此，
希望著名数学家和科学家在此留下足迹，
希望科技界更深刻地感受到数学的作用：
数学强，则科技强；科技强，则国家强！

数学中心官网　　　　数学中心微信公众号

地址：中国湖北武汉珞喻路1037号
华中科技大学创新研究院（恩明楼）8楼
邮编：430074
网页：mathcenter.hust.edu.cn
电邮：mathcenter@hust.edu.cn

Huazhong University of Science and Technology
1037 Luoyu Road, Wuhan, China
Postal Code : 430074
Web : mathcenter.hust.edu.cn
E-mail : mathcenter@hust.edu.cn

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News 新闻

学术活动

报告题目：Stochastic regularization method for linear ill-posed problems

报告人：吕锡亮教授（武汉大学）

报告日期：2024 年 1 月 3 日（星期三）

报告时间：10:00-11:00

腾讯会议：354-175-615

报告摘要：

Due to rapid growth of data sizes in practical applications, in recent years stochastic optimization methods have received tremendous attention and proved to be efficient in various applications of science and technology including in particular the machine learning applications. In this talk we propose randomized Kaczmarz method, stochastic gradient descent method and stochastic mirror descent method for solving linear ill-posed inverse problems. The convergence and convergence rate are provided. Several numerical examples validate the efficiency of the proposed algorithms.

报告人简介：

- 吕锡亮博士，武汉大学数学与统计学院教授。本科毕业于北京大学，并于新加坡国立大学获得硕士、博士学位，曾在马里兰大学、奥地利科学院 RICAM 研究所从事博士后研究，2010 年加入武汉大学数学与统计学院。主要研究方向为反问题理论和计算、机器学习等。

报告题目：**脑启发下的神经网络计算模型研究**
报告人：李秀敏副教授（重庆大学）
报告日期：2024 年 1 月 5 日（星期五）
报告时间：13:30-14:30
腾讯会议：584-470-256

报告摘要：
计算机视觉近年来取得相当进步，然而相较于人类视觉而言，它还存在很大差距。其中关键问题之一在于当前的人工神经网络存在功耗大、依赖带标签大数据样本以及鲁棒性低等问题。鉴于此，为了提升计算机视觉水平，我们将脑科学当前先进成果引入项目交叉研究，提出基于多尺度特征提取机制的视觉皮层神经网络计算模型和图像识别技术，建立由简单（条纹信息）到复杂（局部信息）、由局部到整体、先升维后降维的视觉信息编码和整合策略。利用视觉神经元的朝向选择性初步完成对图像的条纹特征提取；采用突触可塑性学习和动态自平衡调节机制，从高维条纹特征信息进一步提取出低维局部特征，并以稀疏编码的形式存储于网络权值中，构建出一种小样本、低功耗、高鲁棒性图像识别的新型神经网络计算模型，为类脑智能计算提供理论基础。

报告人简介：
- 李秀敏，重庆大学自动化学院副教授，博士毕业于香港理工大学，先后学术访问于剑桥大学的生理学发展与神经科学系及加州大学欧文分校的认知科学系。先后主持国家自然科学基金 1 项、重庆市基础与前沿研究计划面上项目和重庆市技术创新与应用发展专项重点项目各 1 项，参与科技创新 2030-“脑科学与类脑研究”重大项目子课题 1 项等。发表境外 SCI 论文三十余篇。

报告题目：Deep Learning Methods for Parameter Identification in Elliptic Equations: model and error analysis

报告人：焦雨领（武汉大学）

报告日期：2024 年 1 月 16 日（星期二）

报告时间：10:00-12:00

报告地点：恩明楼 813

腾讯会议： 639-783-215

报告摘要：

In this presentation, we introduce a deep learning method for parameter identification in elliptic equations. We begin by establishing novel stability estimates that serve as the guiding principle for proposing appropriate loss functions. We propose a model that leverage Tikhonov regularization and physics-informed neural networks (PINNs). Furthermore, we conduct a rigorous analysis for convergence rates of reconstructions which provide valuable a priori insights for the choice of regularization parameters, as well as the size of the neural networks. Finally, we demonstrate the remarkable stability of the method with respect to the data noise through various numerical experiments.

报告人简介：

- 焦雨领，武汉大学数学统计学院副教授、博导，入选国家高层次人才青年学者计划。主要从事机器学习、科学计算的研究。现任 ACM Transaction on Probabilistic Machine Learning 编委，中国现场统计学会机器学习分会副理事长。相关工作发表在包括 Ann. Stat.、J. Amer. Statist. Assoc.、Statist. Sci.、SIAM J. Math. Anal.、SIAM J. Control Optim.、SIAM J. Numer. Anal.、 SIAM J. Sci. Comput.、 Appl. Comput. Harmon. Anal.、 Inverse Probl.、 IEEE Trans. Inf. Theory、 IEEE Trans. Signal Process.、 J. Mach. Learn. Res.、ICML、 NeurIPS、 AAAI 等期刊和会议上。

基于最优控制理论的最大可能迁移路径检测

Detecting the most probable transition pathway based on optimal control theory

华中科技大学数学中心段金桥教授团队将庞特里亚金极大值原理与逐次逼近格式和嵌套神经网络技术相结合，设计了一种检测随机动力系统最大可能迁移路径的方法。通过在双阱系统、Maier-Stein 化学反应系统和营养物-浮游植物-浮游动物（NPZ）系统三个随机动力系统上验证了该方法，并进行了算法的收敛性分析来说明该方法的有效性。该项工作有助于更好地理解随机波动下复杂系统中的迁移现象。相关论文发表在学术期刊 Applied Mathematical Modelling，论文第一作者为华中科技大学数学中心博士生陈建宇。

研究背景

当外部环境条件（如复杂的噪声）引发系统向新的状态转变时，许多自然系统都会表现出临界迁移现象。因此，检测随机动力系统亚稳态之间最大可能的迁移路径是一个重要的课题。最大可能的迁移路径可以被视为相关的 Onsager-Machlup 作用泛函的极小值。将计算最大可能的迁移路径的变分问题重新表述为确定性最优控制问题。解决最优控制问题的一种有效的方法是通过庞特里亚金极大值原理，但它在高维系统中具有挑战性，因此该工作将神经网络技术与极大值原理相结合，有效求解了高维随机动力系统中的状态迁移路径。

主要研究内容

首先提出一种基于误差反向传播的逐次逼近（Method of Successive Approaximation）方法：

Input: $\theta^0 \in \Theta$, x_0;
Iterations: for k = 0 to K, do;
build neural network for controller θ;
forward solve $\dot{x}_t^{\theta^k} = \nabla_p H\left(t, x_t^{\theta^k}, p_t^{\theta^k}, \theta_t^k\right) = f(t, x_t^{\theta^k}, \theta_t^k)$, $x_0^{\theta^k} = x_0$ with second-order Runge-Kutta;
backward solve $\dot{p}_t^{\theta^k} = -\nabla_x H\left(t, x_t^{\theta^k}, p_t^{\theta^k}, \theta_t^k\right)$, $p_T^{\theta^k} = -\nabla_x \Phi(x_T^{\theta^k})$ with second-order Runge-Kutta;
compute the loss function $L = -\tilde{H}$;
update θ_t^k to $\theta_t^{k+1} = \arg\max_{\theta \in \Theta} \tilde{H}\left(t, X_t^{\theta^k}, P_t^{\theta^k}, \theta, \dot{X}_t^{\theta^k}, \dot{P}_t^{\theta^k}\right)$;
train the network for θ_t^k according to back propagation gradient decent;
after K iterations, the optimal solution is obtained when the loss function converges.
Output: x_t^*, p_t^*, θ_t^* .

上述算法在迭代求解 Hamilton 正则方程时，采用二阶龙格-库塔法。正向求解 x_t^* 同时反向求解伴随变量 p_t^* 。然后建立一个两层的多层感知机（MLP）来训练控制项 θ_t^k 。误差反向传播的最后一个最大化步骤是自动更新神经网络参数。$\tilde{H}$ 表示修正后的哈密顿

量。为了保证算法的收敛性，对原来的哈密顿量进行了修正，提出了如下形式：

$$\tilde{H}\left(t, X_t^*, P_t^*, \theta, \dot{X}_t^*, \dot{P}_t^*\right)$$
$$= H\left(t, X_t^*, P_t^*, \theta\right) - \frac{1}{2}\rho\left\|\dot{X}_t^* - f\left(t, X_t^*, \theta\right)\right\|^2 - \frac{1}{2}\rho\left\|\dot{P}_t^* + \nabla_x H\left(t, X_t^*, P_t^*, \theta\right)\right\|^2$$

其中 ρ 为超参数，实验表明修正后的情形更容易收敛。这种表述的优点是没有明确提及训练数据或目标函数，而是将整个问题表述为控制问题。

之后首先使用经典的一维双阱系统来测试该算法的可行性，并将其与蒙特卡罗模拟结果进行比较，以确保算法的收敛性。然后将该方法应用于二维 Maier-Stein 化学系统，以找到亚稳态之间最大可能的迁移路径。最后将此方法应用于一个三维的营养物-浮游植物-浮游动物（NPZ）系统，从数学的角度提供一些生态学现象的见解。

检测最大可能的迁移路径对于预测随机动力学系统的状态迁移具有重要意义。本项研究已经设计了一种方法来数值求解最大可能的迁移路径，在极小化 Onsager-Machlup 作用泛函的背景下，使用最优控制理论中的庞特里亚金极大值原理和神经网络方法。

论文链接：

https://www.sciencedirect.com/science/article/pii/S0307904X23005607#se0080

基于潜在随机动力系统的预警指标

Early warning indicators via latent stochastic dynamical systems

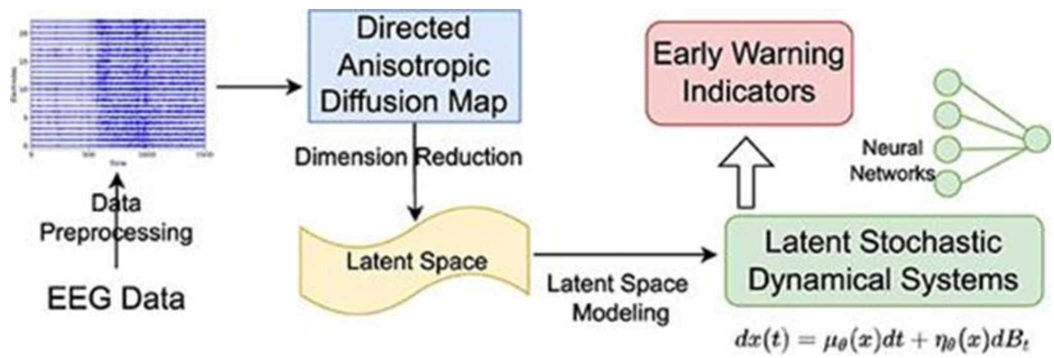
华中科技大学数学中心段金桥教授团队开发了一种新的方法：有向各向异性扩散图，它可捕获低维流形中的潜在动力学。通过降维提取高维数据的有效低维坐标，将潜在动力学与高维数据集联系起来，导出了能够检测临界点的预警信号。相关论文发表在学术期刊 Chaos，论文第一作者为华中科技大学数学中心博士生冯灵羽。

研究背景

在许多现实世界的应用中，如脑疾病、自然灾害和工程可靠性，检测复杂系统或高维观测数据中突然动态转变的预警指标至关重要。

研究内容

潜在动力系统预警信号的工作流程中的主要步骤：



通过潜在坐标和潜在随机动力系统，提取出三种有效的预警信号（Onsager-Machlup 指标、样本熵指标和转移概率指标）。为验证这种方法，研究人员将其应用于癫痫患者的真实数据集。结果发现，这种方法可以有效地检测到状态转换期间的临界点。此外，这种方法还可以自动标记复杂高维时间序列。

研究意义

癫痫发作的早期预警对癫痫患者来说至关重要。在潜在空间中检测突变进行预警，其中正常状态和发作状态可视为两种亚稳定状态。识别亚稳定状态之间转换的能力在预测和控制大脑行为方面发挥着关键作用。利用潜在坐标和潜在随机动力系统的信息，推导出三个有效的预警信号，即 Onsager-Machlup 指标、样本熵指标和转移概率指标。这些指标提高了癫痫发作早期预警系统的稳健性和准确性。此外，从低维数据计算这些指

标的计算成本远低于原始高维数据的计算成本。这种学习潜在随机系统和检测异常动力学的框架有可能扩展到其他复杂高维时间进化数据的一般场景。

论文链接：
https://pubs.aip.org/aip/cha/article/34/3/031101/3268416/Early-warning-indicators-via-latent-stochastic

Academic Achievement 学术成果

数学中心近期研究成果

➢ **付建勋**

正在研究 core Ingram conjecture 相关课题。

➢ **高婷**

1-3 月发表论文：

[1] L Feng, T Gao, W Xiao, J Duan. Early warning indicators via latent stochastic dynamical systems[J]. Chaos: An Interdisciplinary Journal of Nonlinear Science, 2024, 34(3).

[2] J Chen, T Gao, Y Li, J Duan. Detecting the most probable transition pathway based on optimal control theory[J]. Applied Mathematical Modelling, 2024, 127: 217-236.

[3] T Wang, X Wang, Y Shi, W Xin, Z Jiang, T Gao, J Duan. Euler-Maruyama Method Based Channel Prediction: An LDE-Net Implementation and Field Evaluation[J]. IEEE Transactions on Vehicular Technology, 2024.

[4] L Yang, T Gao, W Wei, M Dai, C Fang, J Duan. Multi-task meta label correction for time series prediction[J]. Pattern Recognition, 2024: 110319.

[5] J Guo, T Gao, P Zhang, J Han, J Duan. Deep reinforcement learning in finite-horizon to explore the most probable transition pathway[J]. Physica D: Nonlinear Phenomena, 2024, 458: 133955.

已接收论文：

Fourier neural operator based fluid-structure interaction for predicting the vesicle dynamics

——W Xiao, T Gao, K Liu, J Duan, M Zhao

Accepted by Physica D: Nonlinear Phenomena

提出了一种基于傅里叶神经算子的流固耦合求解器，用于高效模拟 FSI 问题，其中基于有限差分方法的固体求解器与傅里叶神经算子无缝集成，使用浸入边界法预测不可压缩流动，并进行了理论上的收敛性分析以及数值计算模拟。

预印本论文：

Action Functional as Early Warning Indicator in the Space of Probability Measures

——P Zhang, T Gao, J Guo, J Duan

https://arxiv.org/abs/2403.10405

利用 Schordigner 桥理论研究了基于作用泛函的预警指标，并在 Morris-Lecar 模型和真实的阿尔兹海默症数据上进行了分析模拟。

➢ **郁真**

继续基础数学研究。

➢ **刘超**

仍然在研究非线性的 Jeans 不稳定性。目前在一维情况下类锯齿波周期初值下研究 Jeans 不稳定性的一种简化的二阶类波型的模型方程。虽然是模型方程，但是高度的非线性使得问题非常复杂，没有被研究过。目前有一些关键的突破，有望完成该模型的文章。

➢ **林聪萍**

内质网流正反馈模型研究论文已投稿，动态轨道上的非对称拍他模型研究论文已基本完成，近期投稿。

➢ **徐海涛**

利用带有对称性的偏微分方程稳定性理论（GSS）研究格点系统中非线性波的稳定性，讨论连续系统与离散系统的联系。利用 Koopman 算子的线性性质，研究系统的守恒量与讨论可积系统的性质。相关结果正在准备中。

➢ **赵蒙**

Fourier neural operator based fluid-structure interaction for predicting the vesicle dynamic 被 phyisca D 接收。

研究了非同心圆情形下多界面扩张，针对不同的流体粘性和界面的相对位置，研究了界面的动力学和演化过程；还研究了细丝在 Stokes 流中的褶皱现象及其物理机制。

➢ **张一威**

最新研究成果集中在光滑遍历论、几何测度论以及动力系统的概率方法的方面。在与厦门大学吴伟胜教授、华科周小敏副教授的合作中，考虑了条件熵变分公式的一个弱化版本。

发表文章：

W Wu, Y Zhang, X Zhou. Conditional entropy formula with respect to monotonic partitions[J]. Journal of Dynamical and Control Systems, 2024: 1-24.

Qualifying Exams 资格考试

Smooth Manifolds Qualifying Exam 1

Note: This exam covers John M. Lee – “Introduction to Smooth Manifolds”. 10 points for each problem.

1. (a) Let M be a topological manifold with boundary. Prove that:
   i. M is locally compact.
   ii. M is locally path-connected.

   (b) Let M be a topological n-manifold with boundary. Prove that:
   i. ∂M is a closed subset of M and a topological ($n-1$)-manifold without boundary.
   ii. M is a topological manifold if and only if $\partial M = \phi$.

2. Let $P : \mathbb{R}^{k+1} \setminus \{0\} \to \mathbb{R}^{k+1}$ be a smooth function, and suppose that for some $d \in \mathbb{Z}$, $P(\lambda) = \lambda^d P(x)$ for all $\lambda \in \mathbb{R} \setminus \{0\}$ and $x \in \mathbb{R}^{k+1} \setminus \{0\}$. (Such a function is said to be homogeneous of degree d.) Show that the map $\tilde{P} : \mathbb{RP}^k \to \mathbb{RP}^k$ defined by $\tilde{P}([x]) = [P(x)]$ is well defined and smooth.

3. Suppose M, N, P are smooth manifolds with or without boundary, and $F : M \to N$ is a local diffeomorphism. Prove the following:
   (a) If $G : P \to M$ is continuous, then G is smooth if and only if $F \circ G$ is smooth.

   (b) If in addition F is surjective and $G : N \to P$ is any map, then G is smooth if and only if $F \circ G$ is smooth.

4. Suppose $M \subseteq \mathbb{R}^n$ is an embedded m-dim submanifold, and let $UM \subseteq T\mathbb{R}^n$ be the set of all unit tangent vectors to M:
$$UM = \{(x, \upsilon) \in T\mathbb{R}^n : x \in M, \upsilon \in T_x M, |\upsilon| = 1\}$$
   It is called the unit tangent bundle of M. Prove that UM is an embedded $(2m-1)$-dim submanifold of $T\mathbb{R}^n \cong \mathbb{R}^n \times \mathbb{R}^n$.

5. Suppose $F: M \to N$ and $G: N \to P$ are smooth maps, and G is transverse to embedded submanifold $X \subseteq P$. Show that F is transverse to the submanifold $G^{-1}(X)$ if and only if $G \circ F$ is transverse to X.

6. Show that $SO(2)$, $U(1)$ and $\mathbb{S}^1$ are all isomorphic as Lie groups.

7. Let $S \subseteq M$ be a submanifold Let V be a smooth vector field on S. Prove that there exists an open set U containing S and a smooth vector field $\tilde{V}$ on U such that $\tilde{V}|_S = V$.

   Hint: Use partition of unity.

8. Prove that $H^1_{DR}(\mathbb{R}^2) = 0$.

   Hint: Let $w = a(x,y)dx + b(x,y)dy$ be a closed 1-form on $\mathbb{R}^2$. Consider $f(x,y) = \int_0^x a(s,0)ds + \int_0^y b(x,t)dt$.

9. Let M be a smooth manifold with or with or without boundary. Show that the total spaces of TM and T^*M are orientable.

10. Suppose M is an oriented compact smooth manifold with boundary. Show that there does not exist a retraction of M onto its boundary.

    Hint: If the retraction is smooth consider an orientation form on ∂M.

Smooth Manifolds Qualifying Exam 2

Note: This exam covers the book "Introduction to Smooth Manifolds" Chapters 1-16, by John M. Lee.

1. (a) Give the definition of a smooth manifold.

   (b) Show that $\mathbb{RP}^n$ is Hausdorff and second-countable, and is therefore a topological n-manifold.

2. (a) Give the definitions of smooth functions, smooth maps and diffeomorphisms.

   (b) For any topological space *M*; let $C(M)$ denote the algebra of continuous functions $f:M\to\mathbb{R}$. Given a continuous map $f:M\to N$, define $F^*:C(N)\to C(M)$ by $F^*(f)=f\circ F$.

   (i) Show that F^* is a linear map.

   (ii) Suppose *M* and *N* are smooth manifolds. Show that $f:M\to N$ is smooth if and only if $F^*\left(C^\infty(N)\right)\subseteq C^\infty(M)$.

   (iii) Suppose $f:M\to N$ is a homeomorphism between smooth manifolds. Show that it is a diffeomorphism if and only if $F*$ restricts to an isomorphism from $C^\infty(N)\to C^\infty(M)$.

3. Suppose *M* and *N* are smooth manifolds with or without boundary, and $F:M\to N$ is a smooth map. Show that $dF_p:T_pM\to T_{F(p)}N$ is the zero map for each $p\in M$ if and only if *F* is constant on each component of *M*.

4. (a) Give the definitions of submersions, immersions and embeddings.
   Then present some examples.

   (b) State and prove the Inverse Function Theorem for Manifolds.
   State the Rank Theorem.

5. Every smooth *n*-manifold *M* with or without boundary admits a smooth immersion into

$\mathbb{R}^{2n}$ in the special case $\partial M = \emptyset$.

6. If G is a smooth manifold with a group structure such that the map $G \times G \to G$ given by $(g,h) \to gh^{-1}$ is smooth, then G is a Lie group.

7. (a) Define smooth vector fields $X, Y \in \mathfrak{X}(\mathbb{R}^3)$ by

$$X = x\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + x(y+1)\frac{\partial}{\partial z},$$
$$Y = \frac{\partial}{\partial x} + y\frac{\partial}{\partial z}.$$

Compute the Lie bracket $[X,Y]$.

(b) (Extension Lemma For Vector Fields on Submanifolds) Suppose M is a smooth manifold and $S \subseteq M$ is an embedded submanifold with or without boundary. Given $X \in \mathfrak{X}(S)$, show that there is a smooth vector field Y on a neighborhood of S in M such that $X = Y|_S$. Show that every such vector field extends to all of M if and only if S is properly embedded.

8. Let M be a smooth manifold with or without boundary, and let $X : M \to TM$ be a rough vector field. The following are equivalent:
(a) X is smooth.

(b) For every $f \in C^\infty(M)$, the function Xf is smooth on M.

(c) For every open subset $U \subseteq M$ and every $f \in C^\infty(U)$, the function Xf is smooth on U.

(d) Suppose $\omega^1, \ldots, \omega^k$ are linearly independent, and so is the collection of convectors $\eta^1, \ldots, \eta^k \in V^*$.

9. Let M be a smooth manifold with nonempty boundary, and let $\iota : IntM \to M$ denote inclusion. There exists a proper smooth embedding $R : M \to IntM$ such that both $\iota \circ R : M \to M$ and $R \circ \iota : IntM \to IntM$ are smoothly homotopic to identity maps.

Therefore, ι is a homotopy equivalence.

10. (a) Show that $\omega^1 \wedge \ldots \wedge \omega^k$ on a finite-dimensional vector space are linearly dependent if and only if $\omega^1 \wedge \ldots \wedge \omega^k = 0$.

(b) Prove that $span\left(\omega^1, \ldots \omega^k\right) = span\left(\eta^1, \ldots \eta^k\right)$ if and only if there is some nonzero real number c such that $\omega^1 \wedge \ldots \wedge \omega^k = c\eta^1 \wedge \ldots \wedge \eta^k$.

量子力学 Exam 1

1. 质量为 100 克的一块石头以每秒 100 厘米的速度飞行，求它的 *De Broglie* 波长？（提示：普朗克常数约等于 6.6×10^{-34} J • s）

2. 设 $\psi(x)=Ae^{-\frac{1}{2}\alpha^2x^2}$（$\alpha$ 为常数），求归一化常数 A=？（提示：$\int_{-\infty}^{\infty}e^{-y^2}dy=\sqrt{\pi}$ ）

3. 考虑波函数

   $$\Psi(x,t)=Ae^{-\lambda|x|}e^{-i\omega t}$$

   式中 A，λ 和 ω 是正的实数.

   （a）归一化 Ψ，求出 A 的值。

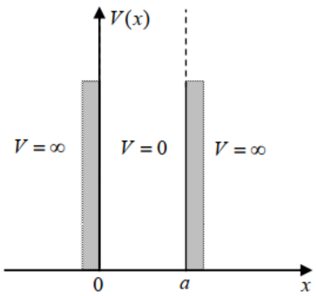
   （b）求出 x 和 x^2 的期待值。

   （c）求出 x 的标准差。

4. 设质量为 m 的粒子在一维无限深势阱中运动，该无限深方势阱表示为

   $$V(x)=\begin{cases}0, 0\le x\le a,\\ \infty,\text{其它地方}\end{cases}$$

   利用定态薛定谔方程求解粒子的定态归一化波函数以及可能的能量值。



5. 接上面第 4 题，试用 de Broglie 的驻波条件，直接求粒子能量的可能取值。

6. 一维谐振子处在基态 $\psi(x)=\sqrt{\frac{\alpha}{\sqrt{\pi}}}e^{-\frac{\alpha^2x^2}{2}-\frac{i}{2}\omega t}$，（提示：$\int_0^{\infty}x^{2n}e^{-ax^2}dx=\frac{1\cdot3\cdot5\cdots\cdots(2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}}$ ）求：

   （1）势能的平均值 $\overline{U}=\frac{1}{2}\mu\omega^2\overline{x^2}$

（2）动能的平均值 $\bar{T}=\dfrac{\overline{p^2}}{2\mu}$

7. 指出下列算符哪个是线性的，说明其理由。

① $4x^2\dfrac{d^2}{dx^2}$

② $[\ \ ]^2$

③ $\sum_{K=1}^{n}$

8. 证明题： 从厄米算符的定义出发，若 Ψ 是 $\hat{F}$ 的属于本征值 λ 的本征函数，即 $\hat{F}\Psi=\lambda\Psi$，证明厄米算符 $\hat{F}$ 的本征值 λ 为实数。

9. 质量为 m 的粒子在势垒内部运动，假设势垒宽度为 a，由测不准关系求出粒子动能的不确定范围。

10. 一个算符 $\hat{A}$ 表示可观测量 A，它的两个归一化本征态是 ψ_1 和 ψ_2，分别对应本征值 a_1 和 a_2。算符 $\hat{B}$ 表示可观测量 B，它的两个归一化本征态是 φ_1 和 φ_2，分别对应本征值 b_1 和 b_2。两组本征态之间有关系：

$\psi_1=(3\varphi_1+4\varphi_2)/5$，$\psi_2=(4\varphi_1-3\varphi_2)/5$.

（a）测量可观测量 A，所得结果为 a_1。那么在测量之后（瞬时）体系处在什么态？

（b）如果现在再测量 B，可能的结果是什么？它们出现的几率是多少？

（c）在恰好测出 B 之后，再次测量 A。那么结果为 a_1 的几率是多少？

量子力学 Exam 2

1. (a) 什么是 de Broglie 波？并写出 de Broglie 波的表达式.
   (b) 什么样的状态是定态？其性质是什么？
   (c) 全同费米子的波函数有什么特点？并写出两个费米子组成的全同粒子体系的波函数.

2. 求质量为 μ 的粒子在势场 $V(x)=\begin{cases}-\dfrac{a}{x}, x>0\\ \infty, x<0\end{cases}$ 中的束缚定态能量与波函数，其中 $a>0$.

3. 在质量为 m 的单原子组成的晶体中，每个原子可看作在所有其他原子组成的球对称势场 $V(r)=\dfrac{1}{2}kr^2$ 中振动，式中 $r^2=x^2+y^2+z^2$. 该模型称为三维各向同性谐振子模型，请给出其能级的表达式.

4. 质量为 μ 的粒子处于一维谐振子势中，在初始时刻的波函数为 $\psi(x,t=0)=A(2\varphi_0+\varphi_1)$.
   (a) 求归一化系数 A.
   (b) 求 $t>0$ 时刻的波函数 $\psi(x,t)$.
   (c) 求 $t=T$ 时刻，体系的能量.
   (d) 求 x 的平均值.

5. (a) 量子力学为什么要用算符表示力学量？并给出厄米算符的定义.
   (b) 表示力学量的算符为什么必须是线性厄米的？

6. 证明 $\left[\hat{p}(x), f(x)\right]=-i\hbar\dfrac{\partial}{\partial x}f(x)$.

7. 已知厄米算符 $\hat{A}$，$\hat{B}$ 满足 $\hat{A}^2=\hat{B}^2=1$，且 $\hat{A}\hat{B}+\hat{B}\hat{A}=0$，求
   (a) 在 A 表象中算符 $\hat{A}$、$\hat{B}$ 的矩阵表示.
   (b) 在 B 表象中算符 $\hat{A}$ 的本征值和本征函数.

(c) 从 A 表象到 B 表象的幺正变换矩阵 S.

8. 氢原子处在基态 $\psi(r,\theta,\varphi)=\frac{1}{\sqrt{\pi a_0}}e^{-\frac{r}{a_0}}$，求

(a) r 的期望值.

(b) 势能 $-\frac{e^2}{r}$ 的期望值.

(c) 最几可的半径.

(d) 动能的期望值.

(e) 动量的概率分布函数.

9. 证明若多粒子系统所受外力矩为 0, 则总角动量 $L=\sum_i I_i$ 守恒.

10. 势能 $V=-\frac{Ze^2}{r}$ 的类氢原子处于 ψ_{nlm} 态，试计算 $\frac{1}{r^2}$ 的平均值. 提示：可利用 Hellmann-Feynman 定理.

量子力学 Exam 3

1. (a) 厄密算符的本征值和本征矢有什么特点？

   (b) 什么样的状态是束缚态、简并态和偶宇称态？

   (c) 全同玻色子的波函数有什么特点？并写出两个玻色子组成的全同粒子体系的波函数。

2. 设全同二粒子的体系的 Hamilton 量为 $\hat{H}(1,2)$，波函数为 $\psi(1,2)$，试证明交换算符 $\hat{P}_{12}$ 是一个守恒量。

3. 一维运动中，哈密顿量 $H=\frac{p^2}{2m}+V(x)$，求 $[x,H]=?$ $[p,H]=?$

4. 质量为 m 的粒子处于如下一维势阱中

$$V(x)=\begin{cases}\infty & (x<0)\\ 0 & (0\le x\le a)\\ V_0(>0) & (x>a)\end{cases}$$

   若已知粒子在此势阱中存在一个能量 $E=\frac{V_0}{2}$ 的本征态，试确定此势阱的宽度 a_0

5. 氢原子在 $t=0$ 时刻处于状态

$$\psi(\hat{r},0)=C\left[\sqrt{\frac{1}{2}}\varphi_1(\hat{r})+\sqrt{\frac{1}{3}}\varphi_2(\hat{r})\sqrt{\frac{1}{2}}\varphi_3(\hat{r})\right]$$

   式中 $\psi_n(\hat{r})$ 为氢原子的第 n 个能量本征态。

   (1) 计算归一化常数 $C=?$

   (2) 计算 $t=0$ 时能量的取值概率与平均值；

   (3) 写出任意时刻 t 的波函数 $\psi(\vec{r},t)$。

$$\sqrt{\frac{3}{8}}\varphi_3(\vec{r})\exp\left(-\frac{i}{\hbar}E_3t\right)$$

6. 质量 m 的粒子，在阱宽为 a 的非对称一维无限深方势阱中运动，当 $t=0$ 时，粒子处于状态 $\psi(x,0)=\frac{1}{2}\varphi_1(x)-\frac{1}{4}\varphi_2(x)+\frac{1}{4}\varphi_3(x)$，其中，$\varphi_n(x)$ 为粒子的第 n 个能量本征态。

   (1) 求 $t=0$ 时能量的取值概率；

   (2) 求 $t>0$ 时的波函数 $\psi(x,t)$；

   (3) 求 $t>0$ 时能量的取值概率。

7. (1) 一粒子的波函数为$\psi(\vec{r})=\psi(x,y,z)$，写出粒子位于$x\sim x+dx$间的几率。

(2) 粒子在一维δ势阱$V(x)=-\gamma\delta(x),(\gamma>0)$，中运动，波函数为$\psi(x)$，写出$\psi'(x)$的跃变条件。

8. 质量为μ的粒子受微扰后，在一维势场中运动，

$$V(x)=\begin{cases}A\cos\dfrac{\pi x}{a},0\le x\le a\\ \infty,x<0,x>a\end{cases}$$

(1) 题中应当把什么看作微扰势？

(2) 写出未受微扰时的能级和波函数；

(3) 用微扰论计算基态能量到二级近似，其中$A=\dfrac{\pi^2\hbar^2}{10\mu a^2}$。

9. 在时间$t=0$时，一个线性谐振子处于用下列归一化的波函数所描写的状态：

$$\psi(x,0)=\sqrt{\frac{1}{5}}u_0(x)+\sqrt{\frac{1}{2}}u_2(x)+c_3u_3(x)$$

式中$u_n(x)$是振子的第n个本征函数。

(1) 试求c_3的数值；

(2) 写出在t时刻的波函数；

(3) 在$t=0$时振子能量的期望值是多少？$t=1$秒时呢？

10. 氢原子处于状态$\psi(\vec{r},s_z)=\begin{pmatrix}\frac{1}{2}R_{21}Y_{11}\\ -\frac{\sqrt{3}}{2}R_{21}Y_{10}\end{pmatrix}=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$，

(1) 求轨道角动量的z分量L_z的平均值；

(2) 求自旋角动量的z分量s_z的平均值；

(3) 求总磁矩$\vec{M}=-\dfrac{e}{2\mu}\vec{L}-\dfrac{e}{\mu}\vec{s}$的$z$分量$M_z$的平均值。

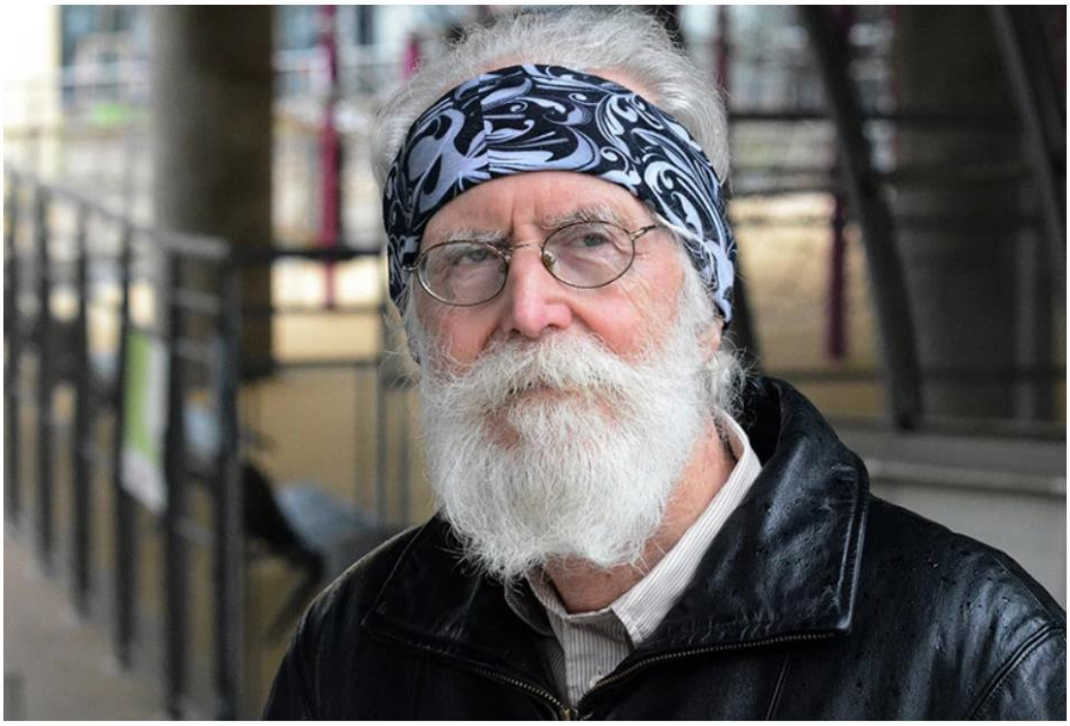
Celebrity Story 名人故事

2024 年阿贝尔奖得主——解释随机性的法国数学家 Michel Talagrand

Mathematician who tamed randomness wins Abel Prize

By Davide Castelvecchi

3 月 20 日，挪威科学与文学院于奥斯陆宣布，2024 年度阿贝尔奖将授予法国数学家 Michel Talagrand，以表彰他在概率论和泛函分析领域的杰出贡献，以及在数学物理和统计学方面的杰出应用。



Michel Talagrand laid mathematical groundwork that has allowed others to tackle problems involving random processes.

A mathematician who developed formulas to make random processes more predictable and helped to solve an iconic model of complex phenomena has won the 2024 Abel Prize, one of the field's most coveted awards. Michel Talagrand received the prize for his "contributions to probability theory and functional analysis, with outstanding applications in mathematical physics and statistics", the Norwegian Academy of Science and Letters in Oslo announced on

20 March.

Assaf Naor, a mathematician at Princeton University in New Jersey, says it is difficult to overestimate the impact of Talagrand's work. "There are papers posted maybe on a daily basis where the punchline is 'now we use Talagrand's inequalities'," he says.

Talagrand's reaction on hearing the news was incredulity. "There was a total blank in my mind for at least four seconds," he says. "If I had been told an alien ship had landed in front of the White House, I would not have been more surprised."

The Abel Prize was modelled after the Nobel Prizes — which do not include mathematics — and was first awarded in 2003. The recipient wins a sum of 7.5 million Norwegian kroner (US$700,000).

'Like a piece of art'

Talagrand specializes in the theory of probability and stochastic processes, which are mathematical models of phenomena governed by randomness. A typical example is a river's water level, which is highly variable and is affected by many independent factors, including rain, wind and temperature, Talagrand says. His proudest achievement was his inequalities1, a set of formulas that poses limits to the swings in stochastic processes. His formulas express how the contributions of many factors often cancel each other out — making the overall result less variable, not more.

"It's like a piece of art," says Abel-committee chair Helge Holden, a mathematician at the Norwegian University of Science and Technology in Trondheim. "The magic here is to find a good estimate, not just a rough estimate."

Thanks to Talagrand's techniques, "many things that seem complicated and random turn out to be not so random", says Naor. His estimates are extremely powerful, for example for studying problems such as optimizing the route of a delivery truck. Finding a perfect solution would require an exorbitant amount of computation, so computer scientists can instead calculate the lengths of a limited number of random candidate routes and then take the average — and Talagrand's inequalities ensure that the result is close to optimal.

Talagrand also completed the solution to a problem posed by theoretical physicist Giorgio Parisi — work that ultimately helped Parisi to earn a Nobel Prize in Physics in 2021. In 1979, Parisi, now at the University of Rome, proposed a complete solution for the structure of a spin glass — an abstracted model of a material in which the magnetization of each atom tends to flip up or down depending on those of its neighbours.

Parisi's arguments were rooted in his powerful intuition in physics, and followed steps that "mathematicians would consider as sorcery", Talagrand says, such as taking n copies of a system — with n being a negative number. Many researchers doubted that Parisi's proof could be made mathematically rigorous. But in the early 2000s, the problem was completely solved in two separate works, one by Talagrand2 and an earlier one by Francesco Guerra3, a mathematical physicist who is also at the University of Rome.

Finding motivation

Talagrand's journey to becoming a top researcher was unconventional. Born in Béziers, France, in 1952, he lost vision in his right eye at age five because of a genetic predisposition to detachment of the retina. Although while growing up in Lyon he was a voracious reader of popular science magazines, he struggled at school, particularly with the complex rules of French spelling. "I never really made peace with orthography," he told an interviewer in 2019.

His turning point came at age 15, when he received emergency treatment for another retinal detachment, this time in his left eye. He had to miss almost an entire year of school. The terrifying experience of nearly losing his sight — and his father's efforts to keep his mind busy while his eyes were bandaged — gave Talagrand a renewed focus. He became a highly motivated student after his recovery, and began to excel in national maths competitions.

Still, Talagrand did not follow the typical path of gifted French students, which includes two years of preparatory school followed by a national admission competition for highly selective grandes écoles such as the École Normale Supérieure in Paris. Instead, he studied at the University of Lyon, France, and then went on to work as a full-time researcher at the national research agency CNRS, first in Lyon and later in Paris, where he spent more than a decade in an entry-level job. Apart from a brief stint in Canada, followed by a trip to the United States where he met his wife, he worked at the CNRS until his retirement.

Talagrand loves to challenge other mathematicians to solve problems that he has come up with — offering cash to those who do — and he keeps a list of those problems on his website. Some have been solved, leading to publications in major maths journals. The prizes come with some conditions: “I will award the prizes below as long as I am not too senile to understand the proofs I receive. If I can’t understand them, I will not pay.”

原文链接：

https://www.nature.com/articles/d41586-024-00839-6

Popular Mathematics 数学热门话题

AI 与数学的融合

随着科技的飞速发展，人工智能（AI）与数学的结合日益紧密，为我们的生活、工作和学习带来了前所未有的变革。在这个时代，AI 与数学的融通共进不仅推动了科技进步，更为人类智慧的发展打开了新的大门。

一、数学是 AI 的基础

数学，作为一门探索数量、结构、变化和空间等概念的学科，自古以来就以其严密的逻辑和精确的计算赢得了人们的广泛赞誉。如今，随着人工智能（AI）的迅猛发展，数学更是成为了这一领域的基石，为其提供了坚实的理论支撑和实践工具。例如，线性代数用于表示向量和矩阵以及处理数据集，概率论和统计学用于建模和理解不确定性和随机性的问题，微积分可用于优化算法，进行数学分析，最优化理论用于解决最优解的问题等等。因此，数学是 AI 发展的重要基础，为其提供了强大的理论支撑的方法工具。

二、AI 的需求推动数学的创新发展

随着 AI 的广泛应用和飞速发展，它所面临的复杂问题和挑战，往往超出了传统数学方法的范畴，从而推动了数学的创新发展。

一方面，AI 对大数据处理的需求推动了数学在统计和概率论方面的创新。传统的统计方法在处理海量数据时显得力不从心，因此，数学研究者开始探索新的统计模型和方法，以适应 AI 的需求。数学研究者开始研究随机优化、分布式优化等新的优化算法满足 AI 对高效、稳定优化算法的需求。

另一方面，AI 的发展也催生了一些新的数学分支和交叉学科。在图像处理中，拓扑学和几何学可以用于形状识别、特征提取等任务；在语音识别中，代数方法可以用于音频信号的分析和处理。深度学习和机器学习也为 AI 在各个领域的应用提供更多新思路和新方法。

AI 的需求推动了数学在多个方面的创新发展。这些创新不仅为 AI 提供了更强大的数学支持，也促进了数学本身的进步和发展。

三、AI 与数学的未来展望

随着 AI 与数学的结合越来越紧密，我们可以预见，未来的科技发展将更加迅猛。一方面，AI 技术将在各个领域得到广泛应用，如医疗、教育、交通等，为人类生活带来更多的便利和福祉。另一方面，数学的发展也将为 AI 技术的进步提供更多的理论支撑和创新思路。

在这个融通共进的时代，AI 与数学的结合不仅推动了科技的进步，更为人类智慧的发展打开了新的大门，AI 与数学的未来展望充满了挑战和机遇。

人工智能物理学家可以推导出想象宇宙的自然法则

An AI physicist can derive the natural laws of imagined universes

MIT 的研究人员开发了一个人工智能系统，该系统被称为“AI 智能物理学家”，它能够推导出一些神秘世界的物理定律，这些神秘世界是为了模拟我们宇宙的复杂性而故意构建的。它标志着创建出不仅能够找到模式，而且可以从这些模式中进行推断，以预测未来的机器学习算法迈出了重要一步，为“人工智能完成科学发现”奠定了基础。

As a student, Galileo famously observed a lamp swinging in Pisa Cathedral and timed its swing against his pulse. He concluded that the period was constant and independent of its amplitude.

Galileo went on to suggest that a pendulum could control a clock and later designed such a machine, although the first clock of this type was built by Huygens some 15 years after Galileo's death.

In making this discovery, Galileo's genius was to ignore all the messy details that were otherwise present in the cathedral—air resistance, temperature, flickering light, noise, other people, and so on. He considered a simple model of a swinging lamp using only its period, focusing on the salient detail.

For many historians, Galileo's approach represents the earliest stage in the evolution of the scientific method, the same process that has produced flight, quantum theory, electronic computing, general relativity, and even artificial intelligence.

In recent years, AI systems have begun to find interesting patterns in data themselves and even derived certain laws of physics as a result. But in these cases, the AI always studied a special data set that had been isolated from real-world distractions. The ability of these AI systems is a long way from the ability of humans such as Galileo.

And that raises an interesting question: is it possible to design an AI system that develops theories the way Galileo did, zeroing in on the information it needs to explain different aspects of the world it observes?

Today we get an answer, thanks to the work of Tailin Wu and Max Tegmark at MIT in Cambridge, Massachusetts. These guys have developed an AI system that copies Galileo's approach and some of the other tricks that physicists have learned over the centuries. Their system—called the AI Physicist—is capable of teasing out several laws of physics in mystery worlds deliberately constructed to simulate the complexity of our universe.

Wu and Tegmark begin by identifying a significant weakness of modern AI systems. When given a big data set, they typically look for a single theory that governs the entire thing. But that becomes increasingly difficult the bigger and more messy the data set becomes.

Indeed, the inside of a cathedral would be a virtually impossible environment for any current AI system to mine for laws of physics.

To cope with this problem, physicists use a number of thought processes to simplify the problem. The first is to develop theories that describe only a small part of the data set. That produces multiple theories that all describe different aspects of the data—like quantum mechanics and relativity, for example.

Wu and Tegmark have developed the AI Physicist to treat big data sets in the same way.

Another general rule that physicists use is Occam's Razor—the idea that simpler explanations are better. That's why physicists generally discount theories requiring a prime mover to create the universe, or the Earth or life itself: the supposed existence of a prime mover raises an additional set of question about its nature and origin.

AI systems are well known for producing overly complex models to describe the data they are trained on. So Wu and Tegmark also teach their system to prefer simpler theories over more complex ones. They do this using a straightforward measure of complexity based on the amount of information the theory encapsulates.

Another famous physicists' trick is to look for ways to unify theories. If one theory can do the job of two, it is probably better. This has driven physicists' quest to find the one law that rules them all (although there is little in the way of actual evidence that such a theory exists).

A final principle that has helped physicists fare well is lifelong learning: the idea that if a particular approach worked in the past, it might work on future problems. So Wu and Tegmark's AI Physicist remembers learned solutions and tries them on future problems.

Armed with these techniques, Wu and Tegmark put their AI Physicist through its paces. They do this by devising 40 mystery worlds governed by laws of physics that vary from one location to another. So a ball thrown into one of these worlds might initially fall under the force of gravity into a region governed by an electromagnetic potential, then into a region governed by a harmonic potential, and so on.

The question that Wu and Tegmark ask is whether their AI Physicist can derive the relevant laws of physics simply by looking at the movement of the ball over time. And they compare the behavior of the AI Physicist with that of a "Newborn Physicist" that uses the same approach but without the benefit of lifelong learning, as well as with a conventional neural network.

It turns out that both the AI Physicist and the Newborn Physicist can derive the relevant laws. "Both agents are able to solve above 90% of all the 40 mystery worlds," they say.

The main advantage of the AI Physicist over the Newborn agent is that it learns more quickly using less of the data. "This is much like an experienced scientist can solve new problems way faster than a beginner by building on prior knowledge about similar problems," say Wu and Tegmark.

And their system is significantly better than a conventional neural network. "Our [AI Physicist] typically learns faster and produces mean-squared prediction errors about a billion times smaller than a standard feedforward neural net of comparable complexity," they say.

That's impressive work that suggests AI systems could have a significant impact on the way science proceeds. Of course, the real test will be to let the AI Physicist loose on a real environment, such as the inside of Pisa Cathedral, and see whether it derives the principle behind mechanical clocks.

Or perhaps to let it loose on other complex data sets, such as those that regularly baffle economists, biologists, and climate scientists. There is surely low-hanging fruit here for a system capable of gathering it.

And if the AI Physicist is successful, historians of science may well look back on it as one of the first steps in a new era of evolution for the scientific method beyond Galileo and his human colleagues. There's no telling where that may take us.

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人工智能如何塑造科学发现

How AI Is Shaping Scientific Discovery

By Sara Frueh

Physicist Mario Krenn sees artificial intelligence as a muse — a source of inspiration and ideas for scientists. It's a description born from his past research and his current work at the Max Planck Institute for the Science of Light, where he and his colleagues develop AI algorithms that can help them learn new ideas and concepts in physics.

His efforts began years ago, when a research team Krenn was part of struggled to come up with an experiment that would let them observe a specific type of quantum entanglement. Krenn, suspecting that their intuition was getting in the way, developed a computer algorithm that can design quantum experiments.

"I let the algorithm run, and within a few hours it found exactly the solution that we as human scientists couldn't find for many weeks," he said. Using the blueprint created by the computer, his colleagues were able to build the setup in the laboratory and use it to observe the phenomenon for the first time.

In a subsequent case, the algorithm overcame a barrier by reviving a long-forgotten technique and applying it in a new context. The scientists were immediately able to generalize this idea to other situations, and they wrote about it in a paper for Physical Review Letters.

"But, if you think about it, none of the core authors of this paper came up with the idea that is described in the paper," said Krenn. "The idea came completely, implicitly from the machine. We were just analyzing what the machine has done."

Krenn was among the speakers at a recent two-day meeting hosted by the National Academies that looked at the present and future of AI in advancing scientific discovery.

AI is advancing science in a range of ways — identifying meaningful trends in large datasets, predicting outcomes based on data, and simulating complex scenarios, said National Academy of Medicine President Victor Dzau in his welcoming remarks. As the technology develops, it may acquire the ability to carry out independent investigations.

“As we envision AI for the future and using it to do independent scientific inquiry, there’s a lot to consider,” said Dzau. “We have to be very careful about understanding the potential of [emerging technologies] possibly affecting society in many different ways … cost, access, equity, ethics, and privacy.” He noted that ongoing committees at NAM are exploring potential impacts in such areas.

Already speeding science

AI is accelerating research on complex neurodegenerative diseases like Alzheimer’s disease and Parkinson’s disease, explained Steven Finkbeiner, a senior investigator at the Gladstone Institutes.

When his team began using AI to analyze images of cells, “one of the very first things that surprised a lot of the biologists in my group was how rich their data might be, and it may contain information that basically we can’t see as humans, or have overlooked,” he said.

His team employed a deep-learning algorithm to try to identify the point at which a cell becomes destined to die — something human scientists have struggled to do, and a key endpoint in understanding neurodegenerative diseases. After being trained with 23,000 examples, the team’s deep-learning network was able to identify changes in the cell nucleus that could predict with high accuracy which cells were destined to die.

Finkbeiner’s team is now using deep learning to identify even earlier changes in a cell that predict its eventual death — early enough that intervening in the process may eventually be possible.

Amy McGovern, a professor at the University of Oklahoma, explained how AI is being applied to meteorology. Initially AI has been used to correct biases in existing weather prediction models, which can improve forecasts and save lives and property.

“Now we are using it to try to improve our foundational understanding of the science of specific events,” she said. For example, researchers are using AI to generate synthetic storms and identify new precursors to tornadoes. Tornadoes are rare enough that real ones alone don't generate enough data to inform that effort.

Autonomy in the future?

Going forward, AI will likely be developed to go beyond tasks like identifying patterns in data and designing experiments. Speakers explored whether there will eventually be "AI scientists" that are able to act independently and also partner with human scientists.

Doing so would mean that AI scientists would have the capacity to perform scientists' core competencies, explained Yolanda Gil, principal scientist at the University of Southern California's Information Sciences Institute. This includes not only tasks like gathering and analyzing data but also a reflection process — what's a good hypothesis to work on? — and the creativity to come up with new paradigms and ideas. "These are big challenges for AI," said Gil.

Hiroaki Kitano, CEO of Sony AI, explained his proposal for the Nobel Turing Challenge — to come up with AI systems by 2050 that can make major discoveries autonomously, at the level of discoveries worthy of a Nobel Prize. "Can AI form a groundbreaking concept that will change our perception?" he asked.

It's both a challenge and a question, Kitano said. "If we manage to build a system like that, is it going to behave like the best human scientists, or does it show a very different kind of intelligence? Are we going to find an alternative form of scientific discovery that is something very different from what we do today?"

Navigating ethical dilemmas

Deborah Johnson, professor emeritus of engineering and society at the University of Virginia, expressed concern about the use of the words "autonomy," "autonomous," and "AI scientist," because they seem to distance human scientists from responsibility for the AI systems they create and any negative impacts that result. "I worry that this is going to lead to a deflection of accountability and responsibility for what happens."

Johnson was on a panel that explored ethical and societal issues that AI research raises —

including how the benefits it yields can be distributed widely rather than reserved for a few.

“Much of the investment and excitement in the areas I work in — in medical artificial intelligence — is about pushing frontiers,” said Glenn Cohen, deputy dean of Harvard Law School. “It’s taking the work of top dermatologists or top brain surgeons and making it even better, helping people who already have access to very high-quality oncology survive longer.”

While that’s great, Cohen continued, much of the benefit of AI lies in its ability to democratize expertise — taking the expertise of average doctors and scaling it up to make it available to people in rural areas and all over the world. Right now, the investment and intellectual property and funding incentives don’t match that ethical goal, and we need to think seriously about how to restructure those incentives, he said.

Vukosi Marivate, ABSA UP Chair of Data Science at the University of Pretoria, said that governance of AI is a team sport; ethical decisions and responsibility shouldn’t rest solely with AI developers and scientists. Society should have a voice in what the expectations for limits on these technologies should be.

“And for society to have a voice, they must understand what is going on,” said Marivate. “It can’t just be that you have these discussions about societal impact, and then society’s not there.” AI developers and scientists should not be making decisions on their own that affect other people broadly, he said.

Moderator Bradley Malin, a professor at Vanderbilt University, emphasized the need to set up an ongoing process to reason about AI-related societal and ethical issues as they inevitably, unpredictably emerge. “These dilemmas are going to arise, and it’s probably unlikely that we’re going to know all of them beforehand.”

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人工智能发现椭圆曲线“杂音”

Elliptic Curve ‘Murmurations’ Found With AI Take Flight

By LYNDIE CHIOU

Mathematicians are working to fully explain unusual behaviors uncovered using artificial intelligence.

Elliptic curves are among the more beguiling objects in modern mathematics. They don’t seem complicated, but they form an expressway between the math that many people learn in high school and research mathematics at its most abstruse. They were central to Andrew Wiles’ celebrated proof of Fermat’s Last Theorem in the 1990s. They are key tools in modern cryptography. And in 2000, the Clay Mathematics Institute named a conjecture about the statistics of elliptic curves one of seven “Millennium Prize Problems,” each of which carries a $1 million prize for its solution. That conjecture, first ventured by Bryan Birch and Peter Swinnerton-Dyer in the 1960s, still hasn’t been proved.

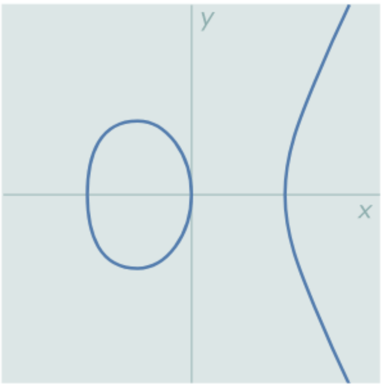
Understanding elliptic curves is a high-stakes endeavor that has been central to math. So in 2022, when a transatlantic collaboration used statistical techniques and artificial intelligence to discover completely unexpected patterns in elliptic curves, it was a welcome, if unexpected, contribution. “It was just a matter of time before machine learning landed on our front doorstep with something interesting,” said Peter Sarnak, a mathematician at the Institute for Advanced Study and Princeton University. Initially, nobody could explain why the newly discovered patterns exist. Since then, in a series of recent papers, mathematicians have begun to unlock the reasons behind the patterns, dubbed “murmurations” for their resemblance to the fluid shapes of flocking starlings, and have started to prove that they must occur not only in the particular examples examined in 2022, but in elliptic curves more generally.

The Importance of Being Elliptic

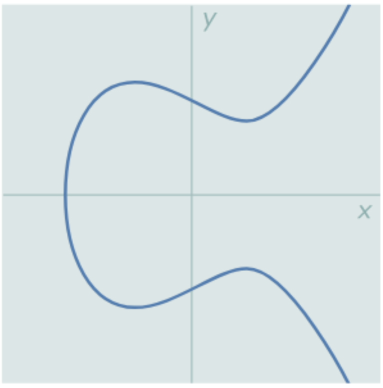
To understand what those patterns are, we have to lay a little groundwork about what elliptic curves are and how mathematicians categorize them.

An elliptic curve relates the square of one variable, commonly written as y, to the third power of another, commonly written as x: $y^2=x^3+Ax+B$, for some pair of numbers A and B, as long as

A and B meet a few straightforward conditions. This equation defines a curve that can be graphed on the plane, as shown below. (Despite the similarity in the names, an ellipse is not an elliptic curve.)



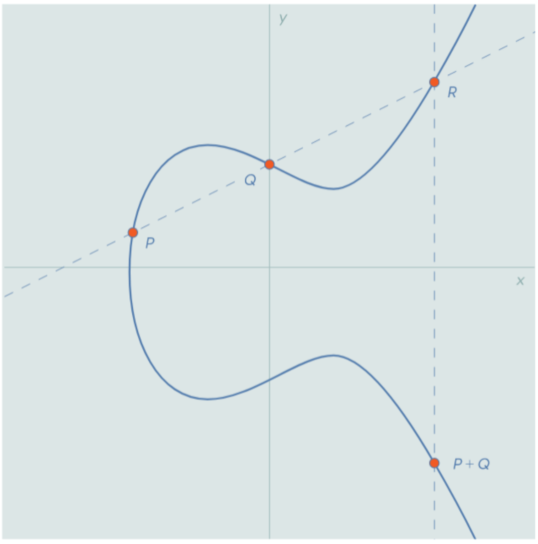
$y^2 = x^3 - x$



$y^2 = x^3 - x + 1$

Though plain-looking, elliptic curves turn out to be incredibly powerful tools for number theorists — mathematicians who look for patterns in the integers. Instead of letting the variables x and y range over all numbers, mathematicians like to restrict them to different number systems, which they call defining a curve "over" a given number system. Elliptic curves restricted to the rational numbers — numbers that can be written as fractions — are particularly useful. "Elliptic curves over the real or complex numbers are quite boring," Sarnak said. "It's only the rational numbers that are deep."

Here's one way that's true. If you draw a straight line between two rational points on an elliptic curve, the place where that line intersects the curve again will also be rational. You can use that fact to define "addition" in an elliptic curve, as shown below.



Draw a line between P and Q. That line will intersect the curve at a third point, R. (Mathematicians have a special trick for dealing with the case where the line doesn't intersect the curve by adding a "point at infinity.") The reflection of R across the x-axis is your sum $P+Q$. Together with this addition operation, all the solutions to the curve form a mathematical object called a group.

Mathematicians use this to define the "rank" of a curve. The rank of a curve relates to the number of rational solutions it has. Rank 0 curves have a finite number of solutions. Curves with higher rank have infinite numbers of solutions whose relationship to one another using the addition operation is described by the rank.

Ranks are not well understood; mathematicians don't always have a way of computing them and don't know how big they can get. (The largest exact rank known for a specific curve is 20.) Similar-looking curves can have completely different ranks.

Elliptic curves also have a lot to do with prime numbers, which are only divisible by 1 and themselves. In particular, mathematicians look at curves over finite fields — systems of cyclical arithmetic that are defined for each prime number. A finite field is like a clock with the number of hours equal to the prime: If you keep counting upward, the numbers start over again. In the finite field for 7, for example, 5 plus 2 equals zero, and 5 plus 3 equals 1.

Patterns formed by thousands of elliptic curves bear a striking similarity to murmurations of starlings. Alex Ramsay/Alamy Stock Photo

An elliptic curve has an associated sequence of numbers, called a_p, which relates to the number of solutions there are to the curve in the finite field defined by the prime p. A smaller a_p means more solutions; a bigger a_p means fewer solutions. Though the rank is hard to calculate, the sequence a_p is a lot easier.

On the basis of numerous calculations done on one of the very first computers, Birch and Swinnerton-Dyer conjectured a relationship between an elliptic curve's rank and the sequence a_p. Anyone who can prove they were right stands to win a million dollars and mathematical immortality.

A Surprise Pattern Emerges

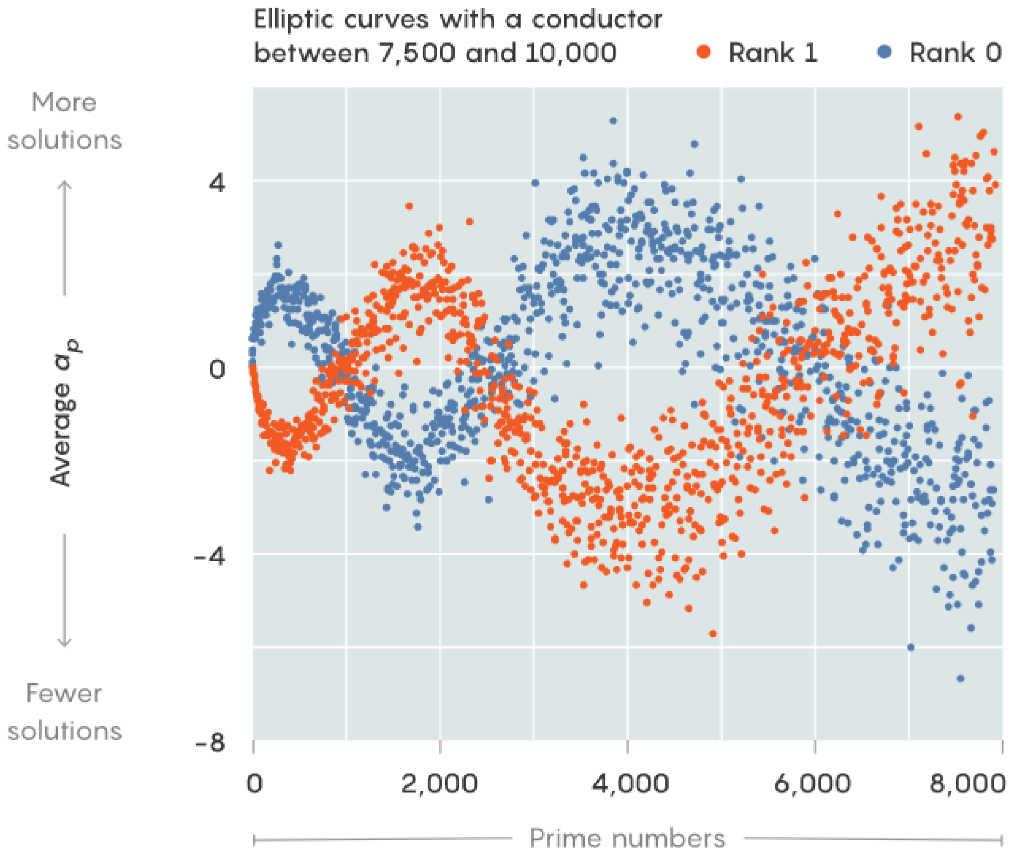
After the start of the pandemic, Yang-Hui He, a researcher at the London Institute for Mathematical Sciences, decided to take on some new challenges. He had been a physics major in college, and had gotten his doctorate from the Massachusetts Institute of Technology in mathematical physics. But he was increasingly interested in number theory, and given the increasing capabilities of artificial intelligence, he thought he'd try his hand at using AI as a tool for finding unexpected patterns in numbers. (He had already been using machine learning to classify Calabi-Yau manifolds, mathematical structures that are widely used in string theory.)

In August 2020, as the pandemic deepened, the University of Nottingham hosted him for an online talk. He was pessimistic about his progress, and about the very possibility of using machine learning to uncover new math. "His narrative was that number theory was hard because you couldn't machine-learn things in number theory," said Thomas Oliver, a mathematician at the University of Westminster who was in the audience. As He remembers, "I couldn't find anything because I wasn't an expert. I was not even using the right things to look at this."

Oliver and Kyu-Hwan Lee, a mathematician at the University of Connecticut, began working with He. "We decided to do this just to learn what machine learning was, rather than to seriously study mathematics," Oliver said. "But we quickly found that you could machine-learn a lot of things."

Oliver and Lee suggested that He apply his techniques to examine L-functions, infinite series closely related to elliptic curves through the sequence a_p. They could use an online database of elliptic curves and their related L-functions called the LMFDB to train their machine learning classifiers. At the time the database had a little over 3 million elliptic curves over the rationals. By October 2020, they had a paper that used information gleaned from L-functions to predict a particular property of elliptic curves. In November they shared another paper that used machine learning to classify other objects in number theory. By December, they were able to predict the ranks of elliptic curves with high accuracy.

But they weren't sure why their machine learning algorithms were working so well. Lee asked his undergraduate student Alexey Pozdnyakov to see if he could figure out what was going on. As it happens, the LMFDB sorts elliptic curves according to a quantity called the conductor, which summarizes information about primes for which a curve fails to behave well. So Pozdnyakov tried looking at large numbers of curves with similar conductors simultaneously — say, all the curves with conductors between 7,500 and 10,000.



This amounted to about 10,000 curves in total. About half of these had rank 0, and half rank 1. (Higher ranks are exceedingly rare.) He then averaged the values of a_p for all the rank 0 curves, separately averaged a_p for all the rank 1 curves, and plotted the results. The two sets of dots formed two distinct, easily discernible waves. That was why the machine learning classifiers had been able to correctly ascertain the ranks of particular curves.

“At first I just felt happy that I’d completed the assignment,” Pozdnyakov said. “But Kyu-Hwan immediately recognized that this pattern was surprising, and that’s when it became really exciting.”

Lee and Oliver were enthralled. “Alexey showed us the picture, and I said it looks like that thing that birds do,” Oliver said. “And then Kyu-Hwan looked it up and said it’s called a murmuration, and then Yang said we should call the paper ‘Murmurations of Elliptic Curves.’”

They uploaded their paper in April 2022 and forwarded it to a handful of other mathematicians, nervously expecting to be told that their so-called “discovery” was well known. Oliver said that the relationship was so visible that it should have been noticed long ago.

Almost immediately, the preprint garnered interest, particularly from Andrew Sutherland, a research scientist at MIT who is one of the managing editors of the LMFDB. Sutherland realized that 3 million elliptic curves weren’t enough for his purposes. He wanted to look at much larger conductor ranges to see how robust the murmurations were. He pulled data from another immense repository of about 150 million elliptic curves. Still unsatisfied, he then pulled in data from a different repository with 300 million curves.

“But even those weren’t enough, so I actually computed a new data set of over a billion elliptic curves, and that’s what I used to compute the really high-res pictures,” Sutherland said. The murmurations showed up whether he averaged over 15,000 elliptic curves at a time or a million at a time. The shape stayed the same even as he looked at the curves over larger and larger prime numbers, a phenomenon called scale invariance. Sutherland also realized that murmurations are not unique to elliptic curves, but also appear in more general L-functions. He wrote a letter summarizing his findings and sent it to Sarnak and Michael Rubinstein at the University of Waterloo.

“If there is a known explanation for it I expect you will know it,” Sutherland wrote.

They didn’t.

Explaining the Pattern

Lee, He and Oliver organized a workshop on murmurations in August 2023 at Brown University's Institute for Computational and Experimental Research in Mathematics (ICERM). Sarnak and Rubinstein came, as did Sarnak's student Nina Zubrilina.

Zubrilina presented her research into murmuration patterns in modular forms, special complex functions which, like elliptic curves, have associated L-functions. In modular forms with large conductors, the murmurations converge into a sharply defined curve, rather than forming a discernible but dispersed pattern. In a paper posted on October 11, 2023, Zubrilina proved that this type of murmuration follows an explicit formula she discovered.

"Nina's big achievement is that she's given a formula for this; I call it the Zubrilina murmuration density formula," Sarnak said. "Using very sophisticated math, she has proven an exact formula which fits the data perfectly."

Her formula is complicated, but Sarnak hails it as an important new kind of function, comparable to the Airy functions that define solutions to differential equations used in a variety of contexts in physics, ranging from optics to quantum mechanics.

Though Zubrilina's formula was the first, others have followed. "Every week now, there's a new paper out," Sarnak said, "mainly using Zubrilina's tools, explaining other aspects of murmurations."

Jonathan Bober, Andrew Booker and Min Lee of the University of Bristol, together with David Lowry-Duda of ICERM, proved the existence of a different type of murmuration in modular forms in another October paper. And Kyu-Hwan Lee, Oliver and Pozdnyakov proved the existence of murmurations in objects called Dirichlet characters that are closely related to *L*-functions.

Sutherland was impressed by the significant dose of luck that had led to the discovery of murmurations. If the elliptic curve data hadn't been ordered by conductor, the murmurations would have disappeared. "They were fortunate to be taking data from the LMFDB, which came pre-sorted according to the conductor," he said. "It's what relates an elliptic curve to the corresponding modular form, but that's not at all obvious. … Two curves whose equations look very similar can have very different conductors." For example, Sutherland noted that $y^2 = x^3 - 11x + 6$ has conductor 17, but flipping the minus sign to a plus sign, $y^2 = x^3 + 11x + 6$ has conductor 100,736.

Even then, the murmurations were only found because of Pozdnyakov's inexperience. "I don't think we would have found it without him," Oliver said, "because the experts traditionally normalize a_p to have absolute value 1. But he didn't normalize them … so the oscillations were very big and visible."

The statistical patterns that AI algorithms use to sort elliptic curves by rank exist in a parameter space with hundreds of dimensions — too many for people to sort through in their minds, let alone visualize, Oliver noted. But though machine learning found the hidden oscillations, "only later did we understand them to be the murmurations."

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華中科技大學数学中心
Center for Mathematical Sciences

Wuhan, China

Web: mathcenter.hust.edu.cn　　E-mail: mathcenter@hust.edu.cn

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欢迎有意愿的学生联系华中科技大学数学中心段金桥主任

（电邮：mathcenter@hust.edu.cn.)

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数学中心网址　　招生视频

地址：中国武汉珞喻路1037号华中科技大学
创新研究院(恩明楼)8楼
邮编：430074
电邮：mathcenter@hust.edu.cn
网页：mathcenter.hust.edu.cn

8th Floor of Enming Building
1037 Luoyu Road, Wuhan, China
Postal Code: 430074
E-mail: mathcenter@hust. edu.cn

文字编辑：刘怡华