



Program Overview

	Morning Session		Afternoon Session I	Afternoon Session II
Saturday June 24	Room 108		Room 205	Room 103
	Chairs: Hongjun Gao, Minghui Song, Meihua Yang		Chairs: Yanbin Tang, Fuke Wu	Chairs: Guanggan Chen, Yayun Zheng
8:45-9:15	Opening Ceremony	14:00-14: 30	Nguyen Dinh Cong	Peter Kloeden
9:15-9:45	Peter Imkeller	14:30-15: 00	Hirofumi Osada	Volker Wihstutz
9:45-10:00	Tea Break	15:00-15: 30	Hans Crauel	Renming Song
10:00-10:3 0	Yuri Kifer	15:30-16: 00	Michael Scheutzow	
10:30-11:0 0	Bjoern Schmalfuss			
11:00-11:3 0	Ludwig Arnold			
11:30-12:0 0				
12:00-14:0 0	Lunch	17:00-19: 00	Conference Dinner	



Sunday June 25	Morning Session		Afternoon Session I	Afternoon Session II
	Room 108		Room 205	Room 103
	Chairs: Xianming Liu, Bjorn Schmalfuss, Ji Li		Chairs: Hongbo Fu, Xiaoyue Li, Xiaopeng Chen	Chairs: Jianhua Huang, Liang Wang, Yejuan Wang
8:45-9:15	Manfred Denker	14:00-14: 30	Jiaowan Luo	Jianhua Huang
9:15-9:45	Yanmei Kang	14:30-15: 00	Zhenxin Liu	Hakima Bessaih
9:45-10:00	Tea Break	15:00-15: 15	Tea Break	
10:00-10:30	Bin Wang	15:15-15: 45	Maria Garrido-Atienza	Jinzhi Lei
10:30-11:00	Youmin Tang	15:45-16: 15	Gerald Trutnau	Hao Ge
11:00-11:30	Wansuo Duan	16:15-16: 45	Gunter Ochs	Jae Kyoung Kim
11:30-12:00	X. San Liang	16:45-17: 15	Cecilia Gonzalez Tokman	Wei Wang
		17:15-17: 45	Vena Pearl Bongolan	Bin Pei
12:00-14:00	Lunch	18:30-20: 30	Dinner	



Monday June 26	Morning Session		Afternoon Session I	Afternoon Session II
	Room 108		Room 205	Room 103
	Chairs: Jifa Jiang, Yiwei Zhang, Ke Yin		Chairs: Xiangjun Wang, Huijie Qiao, Xuwei Ju	Chairs: Zhen Wang, Qinbo Liu, Jintao Wang
8:45-9:15	Anthony Roberts	14:00-14 :30	Caishi Wang	Qigui Yang
9:15-9:45	Huaizhong Zhao	14:30-15 :00	Daiwen Huang	Yejuan Wang
9:45-10:00	Tea Break	15:00-15 :15	Tea Break	
10:00-10:30	Xiaofan Li	15:15-15 :45	Ilya Pavlyukevich	Zhongkui Sun
10:30-11:00	Jifa Jiang	15:45-16 :15	Guanglin Rang	Liang Wang
11:00-11:30	Yongjin Wang	16:15-16 :45	Xiaoliang Wan	Yong Chen
11:30-12:00	Xiaoyue Li	16:45-17 :15	Guangyin Lv	Xiaopeng Chen
		17:15-17 :45	Xiaofeng Ye	Xiang Lv
12:00-14:00	Lunch	18:30-20 :30	Dinner	



Tuesday June 27	Morning Session	Afternoon	
	Room 108 Chairs: Jian Ren, Chujin Li, Xiliang Li	Chairs: Wei Zhang, Jinyue Xu, Yancai Liu, Xiaoli Chen , Pinyuan Wei	
8:45-9:15	Valery Oseledets	Panel Discussion	
9:15-9:45	Peidong Liu		
9:45-10:00	Tea Break		
10:00-10:30	Ale Jan Homburg		
10:30-11:00	Wenxiang Sun		
11:00-11:30	Wen Huang		
11:30-12:00	Yong Xu		
12:00-14:00	Lunch	17:30-19:30	Dinner



Titles and Abstracts

MY CHANCES WITH CHANCE

Recollections and reflections on the occasion of my 80th birthday

Ludwig Arnold

University of Bremen

Abstract: This is a short personal and scientific autobiography. After a few remarks about my childhood in East Germany I describe my “Years of Apprenticeship and Travel” (Germany, USA, Canada 1957-1971). Then I discuss in more detail the “Golden Years” of Random Dynamical Systems in Bremen (1976 ff). Along the way I present some results which I believe are my essential mathematical accomplishments.

On stochastic modified 3D Navier-Stokes equations with anisotropic viscosity

Hakima Bessaih

University of Wyoming

Abstract : Navier-Stokes equations in the whole space subject to an anisotropic viscosity and a random perturbation of multiplicative type is described.

By adding a term of Brinkman-Forchheimer type to the model, existence and uniqueness of global weak solutions in the PDE sense are proved.



These are strong solutions in the probability sense. The convective term given in terms of the Brinkman-Forchheimer provides some extra regularity in the space. As a consequence, the nonlinear term has better properties which allows to prove uniqueness. The proof of existence is performed through a control method. A Large Deviations Principle is also proven.

A Stochastic Agent-based Model for Mangrove Forest Regrowth

Vena Pearl Bongolan

University of the Philippines Diliman

(Joint work with Andrew Vince Lorbisa, Vincent Paul Fiestadaa, Severino G. Salmo IIIb)

Abstract: Mangrove forests along tropical coastlines frequently suffer severe damage due to storms. The mangrove trees are extremely sensitive to environmental stressors such as water salinity and tidal inundation. There is a need for accurate modelling of such systems in order to assist in the decision-making process of environmental conservationists. The authors propose an agent-based model for the prediction of the regenerative behaviour of mangrove stands consisting of the native species and the planted or non-native species in a fragmented habitat, with the use of spatio-temporal coloured noise to simulate stochastic seedling dispersal. It uses Salmo and Juanico's model for individual mangrove growth. This study aims to model accurately the growth of mangrove populations while experiencing said factors and subject to storm damage. Recovery of a population is determined in terms of the current total forest cover compared to the total forest cover before the storm occurred.

Stochastic experiments were carried out in a virtual analogue of Bangrin Marine Protected Area in Bani, Pangasinan with a population of native mangroves (*Avicennia* and *Sonneratia*) and a larger population of planted, non-native mangroves (*Rhizophora*). Out of 1280 runs of various configurations, the native genera always fully recovered within 5.65 to 6.84 years. The planted genera only fully recovered 6 times out of 1280. Within 4.53 to 5.45 years, the planted mangroves were only able to regain around 66.16% to 71.30% of their total pre-storm cover.



Minimal Random Attractors: General Results

Hans Crauel

Goethe-Universität Frankfurt

(Joint work with Michael Scheutzow)

Abstract: Given a random dynamical system and a family of subsets of its state space, an attractor for this family is a compact, strictly invariant set which attracts every element of the family. What attraction is concerned one may distinguish between pullback attraction and forward attraction, both of which imply weak attraction. Common examples are the family of all compact sets or even the family of all bounded sets (yielding set attractors) and all finite sets (yielding point attractors). While a set attractor is unique almost surely, general families of sets to be attracted may fail to have a unique attractor. Always assuming existence of a corresponding attractor, the present work establishes existence of a unique minimal attractor for general families of (possibly non-deterministic) sets both for pullback attraction and for weak attraction. A corresponding result for forward attraction fails to hold true. Michael Scheutzow will in his lecture give an example for the existence of forward attractors none of which is minimal. Also he will provide several examples of minimal pullback and weak attractors.

Well-posedness and large deviations for a class of SPDEs with Levy noise

Yong Chen

Zhejiang Sci-Tech University

Abstract : In this paper, a class of stochastic partial differential equations (SPDEs) with Levy noise is concerned. Firstly, the local well-posedness is established by the iterative approximation. Then the large deviation principle (LDP) for the regularized SPDEs is obtained by the weak convergence approach. To get the LDP for SPDEs here, an exponential equivalence of the



probability measures is proved. The results can be applied to some types of SPDEs such as stochastic Burgers equation, stochastic b-family equation, stochastic modified Novikov equation and stochastic μ -Hunter-Saxton equation.

Stochastic invariant manifolds without gap condition

Xiaopeng Chen

Shantou University

Abstract: The existence of invariant manifolds requires that the spectrum of linear part of a deterministic system or stochastic system contains large gaps. However, for some dynamical systems, the condition of spectral gap is not satisfied. In this case, it is still unknown whether there exists invariant manifolds. This paper concerns the invariant manifolds of the stochastic systems without the spectral gap condition. We present some examples of dynamical systems without the the gap condition and consider the stochastic invariant manifolds of the coresponding stochastic systems.

Generation of nonlocal fractional dynamical systems by fractional differential equations

Nguyen Dinh Cong

Vietnam Academy of Science and Technology

(Joint work with H.T. Tuan)

Abstract: We show that any two trajectories of solutions of a one-dimensional fractional differential equation (FDE) either coincide or do not intersect each other. In contrary, in the higher dimensional case, two different trajectories can meet. Furthermore, one-dimensional FDEs and triangular systems of FDEs generate nonlocal fractional dynamical systems, whereas a higher dimensional FDE does, in general, not generate a nonlocal dynamical system.



An approach to generating mutually independent initial perturbations for ensemble forecasts: orthogonal conditional nonlinear optimal perturbations

Wansuo Duan

University of Chinese Academy of Sciences

(Joint work with Zhenhua Huo)H.T. Tuan

Abstract : Conditional nonlinear optimal perturbation (CNOP) is the initial perturbation that satisfies a certain physical constraint and causes the largest nonlinear evolution at prediction time. To yield mutually independent initial perturbations in ensemble forecasts, orthogonal CNOPs are developed. Orthogonal CNOPs are then applied to a Lorenz-96 model to generate initial perturbations for ensemble forecasting, as compared with orthogonal singular vectors (SVs). When the initial analysis errors are fast-growing, the ensemble forecasts generated by orthogonal CNOPs of the control forecasts perform much more skillfully. Nevertheless, for slow-growing initial analysis errors, the ensemble forecasts generated by orthogonal SVs achieve higher skill when the ensemble initial perturbations are large, whereas the ensemble forecasts generated by orthogonal CNOPs achieve almost the same forecast skill as those generated by orthogonal SVs when the ensemble initial perturbations are sufficiently small. The initial analysis errors that possess much faster growth behavior are easily influenced by nonlinearity, and extreme events (“extreme” here refers to “strong”), due to strong nonlinear instability, may be much more likely to cause fast growth of initial analysis errors. Therefore, the ensemble forecasts generated by orthogonal CNOPs may have higher skill than those generated by orthogonal SVs for extreme events; in particular, the ensemble forecasts generated by orthogonal CNOPs, compared with those generated by orthogonal SVs, require a much small number of ensemble members to achieve high skill. Therefore, orthogonal CNOPs may provide another useful technique to generate initial perturbations for ensemble forecasting.



From random dynamics to partially hyperbolic dynamics

Ale Jan Homburg

University of Amsterdam

Abstract: I'll discuss iterated function systems of diffeomorphisms on compact manifolds with a focus on synchronization by noise and on-off intermittency. The theory will be connected to the seemingly different context of Fubini's nightmare in stably ergodic diffeomorphisms.

Nonequilibrium thermodynamics of chemical reaction kinetics

Hao Ge

Peking University

Abstract: Nonequilibrium thermodynamics and statistical physics in terms of stochastic models entered a stage of vigorous development since 1970s, which well fit the development of advanced experimental techniques in modern physical chemistry and biochemistry. I will discuss our recent stochastic approaches to investigate the nonequilibrium thermodynamics of chemical reaction systems. We show that the entropy production rate can be decomposed into the housekeeping heat and the decreasing rate of relative entropy, both of which are nonnegative, followed by a more stronger version of Clausius inequality. We further proved that in the macroscopic limit by merely allowing the molecular numbers to infinite, a generalized macroscopic free energy and its balance equation emerge. The balance equation is valid generally in isothermal driven systems. A general fluctuation dissipation theorem for stochastic reaction kinetics is also proved. Such an emergent "law" is independent of underlying kinetic details. The mathematical theory illustrates how a novel macroscopic dynamic law can emerge from the mesoscopic kinetics in a multi-scale system.



Dynamics of stochastic fractional Boussinesq equations

Jianhua Huang

National University of Defense Technology

Abstract : In this talk, we present some results on stochastic fractional Boussinesq equations (SFBEs). The existence of random attractor for SFBEs driven by Gaussian noise are established under some suitable assumptions. The global well-posedness of SFBEs driven by Levy noise are obtained. Finally, the existence of mild solution for Caputo-type time fractional stochastic nonlocal Boussinesq equation are also established.

Measure complexity and Mobius disjointness

Wen Huang

Sichuan University

Abstract: We will review some progress about Sarnak's Mobius disjointness conjecture by the measure complexity. Some examples including (Quasi)-discrete spectrum systems and skew product maps on torus over a rotation of the circle will be discussed.

On the Hausdorff dimension of a Weierstrass curve whose components are not controlled

Peter Imkeller

Humboldt University Berlin



(Joint work with G. dos Reis (U Edinburgh) and O. Pamen (U Liverpool and AIMS Ghana))

Abstract: We investigate geometric properties of Weierstrass curves with two components, representing lacunary series based on trigonometric functions.

They are seen to be $\frac{1}{2}$ -Holder continuous, do not possess a Lévy area, and are not (para-)controlled with respect to each other in the sense of our recent Fourier analytic approach of rough path analysis. Their graph is represented as an attractor of a smooth random dynamical system in 4-dimensional Euclidean space. Our argument that the curve is space filling (has Hausdorff dimension 2) is in the spirit of Keller's adaptation of Ledrappier-Young's approach of the Hausdorff dimension of attractors.

On Limiting Behavior of Stationary Measures for Stochastic Evolution Systems with Small Noise Intensity

Jifa Jiang

Shanghai Normal University

(Joint work with Dr. Chen Lifeng and Profs. Dong Zhao and Zhai Jianliang)

Abstract: The limiting behavior of stochastic evolution processes with small noise intensity ϵ is investigated in distribution-based approach. Let μ^ϵ be stationary measure for stochastic process X^ϵ with small ϵ and X^0 be a semiflow on a Polish space. Assume that $\{\mu^\epsilon : 0 < \epsilon \leq \epsilon^0\}$ is tight. Then all their limits in weak sense are X^0 -invariant and their supports are contained in Birkhoff center of X^0 . Applications are made to various stochastic evolution systems, including stochastic ordinary differential equations, stochastic partial differential equations, stochastic functional differential equations driven by Brownian motion or Lévy process. Examples for SODEs are found to show that their limiting measures are supported at saddles.



A Nonconventional Local Limit Theorem and Random Fourier Operators

Yuri Kifer

Hebrew University of Jerusalem

ABSTRACT. We exhibit a local limit theorem for nonconventional sums $S_N = \sum_{n=1}^N F(\xi_n, \xi_{2n}, \dots, \xi_{\ell n})$ where $\{\xi_n\}$ is a Markov chain or it is generated by certain dynamical systems such as mixing subshifts of finite type. The proof leads naturally to the study of iterates of random Fourier operators and to a version of the Ruelle-Perron-Frobenius theorem for them.

Mean-square random dynamical systems and mean-square dichotomies

Peter Kloeden

Huazhong University of Science and Technology

Mean-square random dynamical systems are formulated as nonautonomous dynamical systems on the time-variable state spaces $\in L_2[\Omega, \mathcal{F}_t; \mathbb{R}^d]$, $t \in \mathbb{R}$, of nonanticipative mean-square random variables. They are generated by meanfield stochastic differential equations. Many concepts from nonautonomous dynamical systems carry over, e.g., a mean-square attractor is a pulback attractor. However, lack of compactness criteria for the spaces $L_2[\Omega, \mathcal{F}_t; \mathbb{R}^d]$ make it difficult to apply the theory of nonautonomous dynamical systems. Mean-square dichotomies are defined for linear meanfield SDE and the corresponding spectrum found and is used to illustrate the bifurcation of a zero solution to a nontrivial mean-square attractor.

**The relationship between deterministic and stochastic
quasi-steady-state approximations.**

Jae Kyoung Kim

Korea Advanced Institute of Science and Technology



Abstract: The quasi steady-state approximation (QSSA) is frequently used to reduce deterministic models of biochemical networks. The resulting equations provide a simplified description of the network in terms of non-elementary reaction functions (e.g. Hill functions). Such deterministic reductions are frequently a basis for heuristic stochastic models in which non-elementary reaction functions are used as propensities of Gillespie algorithm. Despite the popularity of this heuristic stochastic simulations, it remains unclear when such stochastic reductions are valid. In this talk, I will present conditions under which the stochastic models with the non-elementary propensity functions accurately approximate the full stochastic models. If the validity condition is satisfied, we can perform accurate and computationally inexpensive stochastic simulation without converting the non-elementary functions to the elementary functions (e.g. mass action kinetics).

Some Quantitative Analysis for Stochastic Dynamics of Gene Regulatory Networks

Yan-Mei Kang

Xi'an Jiaotong University

Abstract: Since the double-helix structure of DNA was discovered in the middle of last century, genetic regulation networks have been designed as digital circuits or devices to emulate the protein-DNA interactions in fundamental cellular processes. In the recent few decades, more and more experimental progresses have been implemented to make the gene regulation networks amenable to quantitative analysis. Here I will talk about some of our work towards this regard.

Genetic regulation networks can be described as differential dynamical systems of stochasticity, non-negativeness and rationality. Note that the state variables in gene regulatory networks represent the concentration of reactants, which cannot physically be negative, so the gene regulatory networks are nonnegative system in general. Due to the Michaelis - Menten or Hill form reaction rate, the genetic regulatory systems are usually governed by differential dynamical systems of rational vector field. The stochasticity of the gene regulatory networks is determined by the essence of biochemical reactions. The process of Gene regulation including transcription and



translation consists of a series of biochemical reactions. Different from the ordinary chemical reactions, the number of the involving reactants or DNA molecules is low and random in the biochemical reactions, thus this sparsity and randomness makes the fluctuations non-negligible and intrinsic in gene regulatory networks. Actually, the significance of randomness has been observed in a plenty of clinical and experimental researches about expression switches, cellular differentiation, phenotypical variety and genetic disease.

Taking the stochasticity and the rationality into account, we take a lambda expression autoregulation model driven by multiplicative and additive noises as example to extend the Gaussian moment method from nonlinear stochastic systems of polynomial vector field to noisy biochemical systems of rational polynomial vector field. Using the extended method, we disclose the phenomenon of stochastic resonance. It is found that the transcription rate can inhibit the stochastic resonant effect, but the degradation rate may take an inverse effect.

Taking the stochasticity and the non-negativeness into account, we model the abnormal fluctuation in gene regulation with the non-Gaussian noise governed by a nonlinear Ornstein-Uhlenbeck process. By imposing an absorbing boundary condition, we analytically deduce the mean first passage times of a gene transcriptional regulatory system driven by additive non-Gaussian noise based on path integral and singular perturbation analysis. We further explore the effect of non-Gaussian noise on stochastic resonance by a novel combination of adiabatic elimination and linear response approximation.

Poisson stable solutions for stochastic differential equations

Zhenxin Liu

Dalian University of Technology

Abstract: In this talk, we will discuss the problem of Poisson stability (in particular stationarity, periodicity, quasi-periodicity, Bohr almost periodicity, Bohr almost automorphy, Birkhoff recurrence, almost recurrence in the sense of Bebutov, Levitan almost periodicity, pseudo-periodicity, pseudo-recurrence, Poisson stability) of solutions for semi-linear stochastic equations by the comparability method.



Causality and information flow within stochastic dynamical systems and causality analysis with time series

X. San Liang

Nanjing Institute of Meteorology

Abstract: Causality analysis is a fundamental problem of interest in a wide variety of disciplines (even in philosophy). In particular, it has been identified as ‘one of the biggest challenges’ in the science of big data. Now, given two time series, can one tell, in a rigorous and quantitative sense, the cause and effect between them? In this talk, we will show that the answer is positive, and, within the framework of stochastic dynamical systems, the formalism can be rigorously established *ab initio*, rather than axiomatically proposed as an *ansatz*. The principle of nil causality that reads, an event is not causal to another if the evolution of the latter is independent of the former, which the classical empirical/half-empirical transfer entropy analysis and Granger causality test fail to verify in many situations, turns out to be a proven theorem here. Formulas in closed form have been obtained for systems such as the baker transformation, Hénon map, stochastic potential flow, to name but a few. For linear stochastic systems, the resulting formula is concise in form, and its maximum likelihood estimator involves only the common statistics namely sample covariance; a corollary is that causation implies correlation, but not vice versa, resolving the long-standing philosophical debate over causation versus correlation. A news report in *PhysicsToday* is referred to the link below.

The above formula has been validated with touchstone series purportedly generated with one-way causality that defies the classical approaches such as Granger causality test and transfer entropy analysis. It has also been applied successfully to the investigation of a variety of real problems. Through a simple analysis with the stock series of IBM and GE, an unusually strong one-way causality is identified from the former to the latter in their early era, revealing to us an old story, which has almost faded into oblivion, about “Seven Dwarfs” competing with a “Giant” for the computer market.

Another example presented here regards the cause-effect relation between the two climate modes, El Niño and Indian Ocean Dipole (IOD). In the third example, an unambiguous one-way causality is found between CO₂ and



the global mean temperature anomaly. While it is confirmed that CO₂ indeed drives the recent global warming, on paleoclimate scales the cause-effect relation may be completely reversed. Also will be presented are the identification of a simple pattern underlying a chaotic attractor, and a preliminary study of the causal structure in the near-wall turbulence.

On Stochastic Multi-Group Lotka-Volterra Ecosystems with Regime Switching

Xiaoyue Li

Northeast Normal University

Abstract: Focusing on stochastic dynamics involving continuous states as well as discrete events, this paper investigates dynamical behaviors of stochastic multi-group Lotka-Volterra model with regime switching. The contributions of the paper lie on: (a) giving the sufficient conditions of stochastic permanence for generic stochastic multi-group Lotka-Volterra model, which are much weaker than the existing results in the literature; (b) obtaining the stochastic strong permanence and ergodic property for the mutualistic systems; (c) establishing the almost surely asymptotic estimate of solutions. These can explain some recurring phenomena in practice and reveal the fact that regime switching can suppress the impermanence. A couple of examples and numerical simulations are given to illustrate our results.

Numerical simulations of macroscopic quantities for stochastic differential equations with alpha-stable processes

Xiaofan Li

Illinois Institute of Technology, USA

Abstract : The mean first exit time, escape probability and transitional probability density are utilized to quantify dynamical behaviors of stochastic



differential equations with non-Gaussian, α -stable type Lévy motions. Taking advantage of the Toeplitz matrix structure of the time-space discretization, a fast and accurate numerical algorithm is proposed to simulate the nonlocal Fokker-Planck equations on either a bounded or infinite domain. Under a specified condition, the scheme is shown to satisfy a discrete maximum principle and to be convergent. The numerical results for two prototypical stochastic systems, the Ornstein-Uhlenbeck system and the double-well system are shown.

Differentiation of SRB measures for hyperbolic attractors under random perturbations

Peidong Liu

Peking University

(Joint work with Kening Lu and Zeng Lian)

Abstract: Linear response formulas arise from needs in statistical mechanics. We will give an introduction to a general but formal linear response formula (mainly due to Ruelle) in dynamical systems, and then discuss the cases of hyperbolic attractors and their random perturbations.

Small-gain theorems for nonlinear stochastic systems with inputs and outputs II: Multiplicative white noise case.

Xiang Lv

Shanghai Normal University

(Joint work with Prof. Jifa Jiang)

Abstract: This paper is a continuation of the paper [SIAM J. Control Optim., 54 (2016), pp. 2383–2402], which focuses on exploring the global stability of nonlinear stochastic feedback systems on the nonnegative orthant driven by



multiplicative white noise and presenting a couple of small-gain results. We investigate the dynamical behavior of pull-back trajectories for stochastic control systems and prove that there exists a unique globally attracting positive random equilibrium for those systems whose output functions either possess bounded derivatives or are uniformly bounded away from zero. Our results can be applied to well-known stochastic Goodwin negative feedback system, Othmer-Tyson positive feedback system and Griffith positive feedback system as well as other stochastic cooperative, competitive and predator-prey systems.

The Erdos measures on Euclidean space, on \widehat{Z}^n and Heisenberg group

Valery Oseledets

Financial University and Moscow State University

We study the distribution of the random variable :

$$\zeta = A^{-1}\xi_1 + A^{-2}\xi_2 + \dots,$$

where $\xi_k \in Z^n$ are independent identically distributed random variables, $0 < P(\xi_1 = 0) < 1$, the expanding matrix $A \in M_n(Z)$.

We will call the distribution of the random variable ζ the *Erdős measure* on the space R^n . We will call the distribution of the random variable ζ the *Erdős measure* on the space R^n . This measure is some self-similar measure.

Another measure is the distribution of the random variable:

$$\widehat{\zeta} = \xi_0 + A\xi_1 + A^2\xi_2 + \dots,$$

Here $\widehat{\zeta} \in \widehat{Z}^n$, where \widehat{Z}^n is the profinite completion of Z^n with respect to $Z^n > AZ^n > A^2Z^n > \dots$.

We will call the distribution of the random variable ζ the *Erdős measure* on the group \widehat{Z}^n .

For Heisenberg group we consider the distribution of the random variable

$$\zeta_H = \prod_{k=1}^{\infty} \Phi^{-k} \xi_k,$$

where $\xi_k \in L$ are independent identically distributed random variables, $0 < P(\xi_1 = Id) < 1$, L is discrete Heisenberg group, Φ is expanding automorphism of Heisenberg group and $\Phi(L) < L$.

We will call the distribution of the random variable ζ_H the *Erdős measure* on Heisenberg group.

We obtain a necessary and sufficient condition for the absolute continuity of these distributions.

We use the notions of invariant *Erdős measure* on the torus, the invariant *Erdős measure* on the compact abelian group \widehat{Z}^n and the invariant *Erdős measure* on Heisenberg nilmanifold.

We show a connection of these invariant measures with the functions of countable stationary Markov chains. In the case when $|\{j : p_j \neq 0\}| < \infty$ we establish the relation of these invariant measures to the case of finite stationary Markov chains.



Some ideas on chaotic random attractors.

Gunter Ochs

Hochschule Darmstadt University of Applied Sciences

Abstract: We consider the situation of a random dynamical system with a global attractor which contains a deterministic fixed point. If this fixed point has a positive and a negative Lyapunov exponent, previous case studies of the stochastic Duffing-van der Pol oscillator by Keller and Ochs have indicated some sort of chaotic dynamics possibly due to random homoclinic points.

In this talk we present an approach to find general conditions, under which it is possible to prove transversal intersections between stable and unstable manifolds of the fixed point, which leads to chaotic dynamics. In addition to our theoretical considerations we will show numerical examples.

Stochastic Averaging for A Class of Two-Time-Scale Systems of Stochastic Partial Differential Equations.

Bin Pei

Northwestern Polytechnical University

(Joint work with Yong Xu, George Yin)

Abstract: We focus on systems of stochastic partial differential equations that have a slow component driven by a fractional Brownian motion and a fast component driven by a fast-varying diffusion. We establish an averaging principle in which the fast-varying diffusion process acts as a "noise" and is averaged out in the limit. The slow process is shown to have a limit in the mean sense, which is characterized by the solution of a stochastic partial differential equation driven by a fractional Brownian motion whose coefficients are averages of that of the original slow process with respect to the stationary



measure of the fast-varying diffusion. This averaging principle paves a way for reduction of computational complexity. The implication is that one can ignore the complex original systems and concentrates on the average systems instead.

**Small noise behaviour of non-linear Langevin equations driven by
symmetric Levy noise**

Ilya Pavlyukevich

Friedrich-Schiller-Universität Jena

(Joint work with Alexei Kulik, Kiev)

Abstract: We consider a second-order Langevin equation for the motion of a Levy particle subject to a non-linear friction force being a power of the particle's velocity, $F = -|v|^\beta \text{sign}(v)$. In the limit of the small noise amplitude we determine the law of the displacement process.

**Couple periodic-patches of a stochastic microscale to predict emergent
macroscale dynamics**

Anthony Roberts

University of Adelaide

Abstract : Molecular simulations, while in principle deterministic on the microscale, are chaotic and so effectively stochastic on meso/macro-scales. Many other microscale simulations are directly stochastic. Furthermore, as for many microscale systems, molecular simulations are much the easiest to code with periodic boundary conditions in space. We seek to distribute many small patches of such stochastic microscale periodic simulators over a macroscale grid. Then the research aim is find a coupling between the small periodic-patches in order to effectively and



cheaply simulate the macroscale dynamics. A stochastic slow manifold analysis of the system of stochastic periodic-patches indicates that a proportional control applied within action regions of each patch can be effective. The analysis suggests Lagrange interpolation of core-patch averages provides a useful control. A pilot implementation of the coupling to a small triply-periodic patch of atomistic simulation indicates that the scheme usefully predicts the emergent heat diffusion.

Local stability of stochastic differential equations driven by fbm with Hurst parameter $H \in (1/3, 1/2]$

Bjoern Schmalfuss

Friedrich-Schiller-Universität

Abstract: We use the method of compensated derivatives to formulate and to solve an SDE with Hurst parameter $H \in (1/3, 1/2]$. In addition we formulate conditions on the coefficients such that the solution of this equation is locally exponential stable.

Minimal Random Attractors: Examples and Counterexamples

Michael Scheutzow

Technische Universität Berlin

(Joint work with Hans Crauel)

Abstract: We complement the results on the existence of minimal random attractors in the talk by Hans Crauel with several examples. We provide an example showing that a minimal forward attractor may not exist in spite of the fact that (many) forward attractors exist. We also show that a minimal



pullback point attractor may be significantly different from the closure of the union of all Omega-limit sets of all singletons.

Potential theory of subordinate killed Levy processes

Renming Song

University of Illinois

Abstract: Let Z be a subordinate Brownian motion in \mathbb{R}^d , $d \geq 3$, via a subordinator with Laplace exponent φ . We kill the process Z upon exiting a bounded open set $D \subset \mathbb{R}^d$ to obtain the killed process Z^D , and then we subordinate the process Z^D by a subordinator with Laplace exponent ψ . The resulting process is denoted by Y^D . Both φ and ψ are assumed to satisfy certain weak scaling conditions at infinity.

We study the potential theory of Y^D , in particular its boundary theory. If D is a smooth open set, we establish sharp two-sided estimates on the Green function of Y^D and prove the boundary Harnack principle with explicit decay rate near the boundary of D . These results show that the boundary behavior of Y^D is completely determined by φ . On the other hand, in the interior of D , the behavior of Y^D is determined by the composition $\psi \circ \varphi$. Under an additional condition on ψ , we also establish sharp two-sided estimates on the jumping kernel of Y^D .

In case that D is a δ -fat bounded open set, we show that the Harnack



inequality holds, and prove a boundary Harnack principle for non-negative functions harmonic in a smooth open set E strictly contained in D . If, in addition, D satisfies the local exterior volume condition, then we show that the Carleson estimate holds.

Periodic orbits and atomic measures on non-uniformly hyperbolic systems

Wenxiang Sun

Peking University

Abstract: A hyperbolic measure and its Lyapunov exponents can be approximated by atomic measures and their Lyapunov exponents respectively.

Progress towards improving seasonal climate prediction by mathematical methods.

Youmin Tang

University of Northern British Columbia

Abstract: In this talk, we will present some progresses in improving seasonal climate predictions by using more advanced mathematical methods. The first example is to rely on the basic properties of stochastic theory to develop an efficient technique for the extraction of climatically relevant singular vectors (CSV) in the presence of weather noise. Emphasis is placed on the applications of the CSV in seasonal climate predictions and to construct optimal ensemble climate predictions. The results indicate that the CSVs can well characterize the optimal error growth of the climate predictions and lead to better ensemble predictions than traditional time lag (TLE) method. The second example is about our recent progress in the state estimate of state-space models with applications of Bayesian-based algorithms. A



simplified algorithm of Sigma-point Kalman filter is develop to deal with the state estimation of high-dimensional systems like atmospheric and oceanic general circulation models.

Existence and uniqueness results for Ito-SDEs with general drifts and

Sobolev diffusion coefficients

Gerald Trutnau

Seoul National University

(Joint work with Haesung Lee (Seoul National University) and Minjung Gim (NIMS, South Korea)).

Abstract: Using the Dirichlet form method, we construct weak solutions to SDEs in Euclidean space with general measurable coefficients up to their explosion times. Restricting the assumptions on the coefficients to the ones of [Zhang, X., Strong solutions of SDES with singular drift and Sobolev diffusion coefficients, SPA 2005] or [Krylov, N. V., Rockner, M., Strong solutions of stochastic equations with singular time dependent drift, PTRF 2005], we obtain by the uniqueness results of the mentioned papers that our weak solutions coincide with the pathwise unique and strong solutions constructed there up to their explosion times. The use of our approach is that we can now additionally use analytic non-explosion criteria to obtain pathwise unique and strong solutions up to infinity. In particular, analytic non-explosion and recurrence criteria will be presented.

A spectral approach for quenched limit theorems for random dynamical systems

Cecilia Gonzalez Tokman



The University of Queensland

(Joint work with D. Dragicevic, G. Froyland and S. Vaienti)

Abstract: Random or non-autonomous dynamical systems provide very flexible models for the study of forced or time-dependent systems, with driving mechanisms allowed to range from deterministic forcing to stationary noise. In this talk, we present a spectral approach to the study of non-autonomous dynamics, developed in the last decade via multiplicative ergodic theory. We then show how spectral methods can be used to establish quenched limit theorems for a class of non-autonomous dynamical systems.

Noise induced rotation versus resonance in randomly perturbed linear systems.

Volker Wihstutz

University of North Carolina at Charlotte

Abstract: Surveying older and newer results on physically realizable linear systems with multiplicative noise, we consider multiplicative noise as being able – like an outer force to produce resonance in the unperturbed system. We look after noise generated resonance versus non-resonance and noise induced rotation or combinations of both occurring, in particular, in systems of higher finite dimensions and infinite dimensions.

Efficient minimum action method for capturing the most probable transition path

Xiaoliang Wan

Louisiana State University

Abstract : Minimum action method (MAM) plays an important role in minimizing the Freidlin-Wentzell action functional, which is the central object of



the Freidlin-Wentzell theory of large deviations for small-noise-induced transitions in stochastic dynamical systems. Because of the demanding computation cost, especially in spatially extended systems, numerical efficiency is a critical issue for MAM. Difficulties come from both temporal and spatial discretizations. One severe hurdle for the application of MAM to large scale systems is the global reparametrization in time direction, which is needed in most versions of MAM to achieve accuracy. We have developed a new version of MAM based on optimal linear time scaling where the global reparametrization is replaced by hp adaptivity guided by a posteriori error estimates. More specifically, we use the zero-Hamiltonian constraint to define an indicator to measure the error induced by linear time scaling, and the derivative recovery technique to construct an error indicator and a regularity indicator for the transition paths approximated by finite elements.

On a class of stochastic differential equations with local times adjustment

Yongjin Wang

Nankai University

Abstract : In this talk, we shall address a class of stochastic differential equations with local times adjustment, which actually conduct a class of stochastic processes with partial reflections, also called skew diffusion processes. We start at the skew Brownian motion, and describe its construction and the corresponding SDE with local time. Next we are going to study skew Ornstein-Uhlenbeck processes and skew Feller branching processes. We are going to derive their probability transition densities expressions and the Laplace transforms of the first Hitting times. Those quantities are playing fundamental roles in the analysis of the stochastic processes.



Multi-valued non-autonomous random dynamical systems

Yejuan Wang

Lanzhou University

Abstract : We first present a sufficient and necessary condition for the existence of pullback attractors of multi-valued non-compact random dynamical systems. We then prove the existence of pullback attractors for reaction-diffusion equations with non-autonomous deterministic as well as stochastic forcing terms for which the uniqueness of solutions need not hold. In particular, the existence of extremal complete trajectories is established for the multi-valued non-compact random dynamical system. Finally, the asymptotic behavior of stochastic systems with a Caputo fractional time derivative is investigated, and the existence of a global forward attracting set in the mean-square topology is given.

Stochastic Schrodinger Equations Associated with Bernoulli Annihilation

and Creation Operators

Caishi Wang

Northwest Normal University

(Joint work with Jinshu Chen)

Abstract: Bernoulli annihilation and creation operators are annihilation and creation operators acting on Bernoulli functionals, which satisfy a canonical anti-commutation relation (CAR) in equal-time. In this talk, we consider linear stochastic Schrodinger equations (LSSE) associated with Bernoulli annihilation and creation operators in the space of square integrable Bernoulli functionals. Among others, we obtain an easily-checking condition for such an equation to have a unique N -strong solution of mean square norm conservation for given N , where N is the number operator acting on Bernoulli functionals.



A diffusion approximation for random nonlinear wave equation with large interaction

Wei Wang

Nanjing University

Abstract: This talk presents a diffusion approximation for random nonlinear wave equation with large interaction and small mass. The limit is shown to be a one dimensional one order stochastic differential equation. The boundary with a randomly high oscillation is also considered.

Data assimilation scheme for decadal predictions using a coupled climate system model

Bin Wang

Tsinghua University & Chinese Academy of Sciences

(Joint work with Yujun He, Mimi Liu)

Abstract: The initialization for decadal predictions using a coupled model includes a long-term cycle of data assimilation in the coupling framework, which is an essential step of decadal prediction. The skill of decadal prediction closely relies on the performance of the assimilation scheme used in the initialization. Most decadal prediction experiments conducted in the Coupled Model Intercomparison Project Phase 5 (CMIP5) simply used the nudging approach, and very few applied the variational or ensemble methods that are very expensive in computing. Therefore, it is necessary to develop an assimilation scheme that can well use the observations in an economical way. In this paper, we propose a coupled assimilation scheme that can incorporate 4-dimensional ocean observations (say temperature and salinity) into the IC of ocean component model and in the meantime can be time-saving. Numerical experiments using a coupled climate system model are conducted to test the performance of the new assimilation scheme.



The response analysis of fractional-order stochastic system via generalized cell mapping method

Liang Wang

Northwestern Polytechnical University

(Joint work with Lili Xue)

Abstract: We research the response of the fractional-order stochastic system via generalized cell mapping method. The short memory principle is introduced to ensure that the response of the system is a Markov process. The generalized cell mapping method is applied to display the global dynamics of the noise-free system. The stochastic generalized cell mapping method is employed to obtain the evolutionary process of probability density functions of the response. The fractional-order Duffing system and the SD oscillator with fractional derivative damping are taken as examples to give the implementations of our strategies discussed above. Studies have shown that the evolutionary direction of PDF of fractional-order stochastic system is consistent with the unstable manifold. We also found that the smoothness parameter, the noise intensity, the amplitude and frequency of the harmonic force can induce the occurrence of stochastic P-bifurcation in the SD oscillator. The effectiveness of the method is confirmed using Monte Carlo simulations.

Random attractors for stochastic differential equations driven by two-sided Lévy processes

Yong Xu

Northwestern Polytechnical University

(Joint work with Xiaoyu Zhang and Bjoern Schmalfuss)

Abstract: At present, the study of random attractors mainly concentrate on the stochastic differential equations driven by Gaussian process. However, there are much complex phenomena perturbed by non-Gaussian noise especially Lévy noise in practice. Lévy process, as a typical type of non-Gaussian



process with jumps, can describe the discontinuous complex phenomena with big jumps accurately. Up to now the research on the relevant basic theories of random attractors of the systems driven by Lévy process has far-reaching implications since the stochastic integral should be re-defined. In this talk, the random attractors and random bifurcations in random dynamical systems which are solutions of nonlinear Marcus type stochastic differential equations with multiplicative Lévy noise are investigated.

Random Transformations and its applications to Synchronization

Xiaofeng Ye

University of Washington

Abstract: We establish some elementary contradistinctions between Markov chain (MC) and RDS descriptions of a stochastic dynamics. It is shown that a given MC is compatible with many possible RDS, and we study in particular the corresponding RDS with maximum metric entropy. Specifically, we show an emergent behavior of an MC with a unique absorbing and aperiodic communicating class, after all the trajectories of the RDS synchronizes.

We also consider a finite state hidden Markov model (HMM) with multidimensional observations. Under some mild assumptions, the prediction filter forget almost surely the initial condition exponentially fast. However, it is very difficult to calculate this asymptotic rate of exponential loss of memory analytically. We restate this problem as synchronization in random dynamical system and we use the Lyapunov exponents of the induced random dynamical system defined in the projective space \mathbb{R}^{n-1} to approximate the convergence rate. Finally, we propose a stable numerical algorithm to calculate the rate of exponential forgetting. The numerical simulation result and comparison with current upper bound in literature will be shown in the presentation.



Periodic Stochastic Dynamical Systems (PeriSDS)

Huaizhong Zhao

Loughborough University

(Joint work with C Feng)

Abstract: I will talk random periodic processes, periodic measures and their “equivalence” . I will also talk about their existence, ergodic theory and pure imaginary eigenvalues of the corresponding Markovian semigroup.
